REDSPOT

O LEVEL

# CLASSIFIED WORKED SOLUTIONS

# MATHEMATICS

## (Paper 1 - All Variants)

**(Syllabus 4024)**

| | |
|---|---|
| period | **2015 to 2024** |
| contents | **June & November, Paper 1 (P11 & P12) Worked Solutions** |
| form | **Topic By Topic** |
| compiled for | **O Levels** |



Tel No : 042-35201010
Mobile No : 0300-8447654
E-Mail : info@redspot.com.pk
Website : www.redspot.pk
Address : P.O. Box 5041, Model Town, Lahore, Pakistan.

*Appointed Agents & Wholesalers in PAKISTAN:*

**BOOK LAND**
Urdu Bazaar, Lahore. Tel: 042-37124656

**NATIONAL BOOK SERVICE**
Urdu Bazaar, Lahore. Tel: 042-37247310.

**LAROSH BOOKS**
Urdu Bazaar Lahore. Tel: 042-37312126.

**BURHANI BOOK CENTRE**
New Urdu Bazar, Karachi, Tel: 021-32634718

**MARYAM ACADEMY**
Urdu Bazaar, Karachi, Tel: 0331-2425264

**TARIQ BOOK TOWN**
Samar Garden, Hydari North nazimzbad, Karachi. Tel: 021-34555918, 36649425

**REHMAN BOOKS**
College Road, Rawalpindi
Tel: 051-5770603, 0313-5618976

**WELCOME BOOKS**
Soneri Masjid Road, Peshawar Cantt.
Tel: 091-5274932, 0300-5860068

# CONTENTS

# CONTENTS

# TOPIC 1

# Numbers

**1.** (a) Evaluate $\frac{3}{8}+1\frac{1}{3}$.

*Answer* ..... [1]

(b) Evaluate $5-3(2-1.4)$.

*Answer* ..... [1]

*[June/2015/P11/Q1]*

---

**2.** Arrange these fractions in order, beginning with the smallest.

$$\frac{4}{5} \quad \frac{3}{4} \quad \frac{31}{40}$$

*Answer* ..... , ..... , ..... [1]
*smallest*

*[June/2015/P11/Q3(b)]*

---

**3.** (a) Evaluate $\frac{1.3+2.9}{0.2}$.

*Answer* ..... [1]

(b) Evaluate $2\frac{1}{4}\times\frac{1}{5}$.

*Answer* ..... [1]

*[June/2015/P12/Q1]*

---

**4.** Write these numbers in order of size, starting with the smallest.

$$\frac{13}{20} \qquad 0.7 \qquad \frac{7}{12} \qquad 0.64 \qquad \frac{5}{8}$$

*Answer* ..... , ..... , ..... , ..... , ..... [2]
*smallest*

*[June/2015/P12/Q2]*

---

**5.** (a) Express 60 as a product of its prime factors.

*Answer* ..... [1]

(b) Find the smallest possible integer $m$ such that $60m$ is a square number.

*Answer* $m$ = ..... [1]

(c) The lowest number that is a multiple of both 60 and the integer $n$ is 180 .
Find the smallest possible value of $n$.

*Answer* $n$ = ..... [1]

*[June/2015/P12/Q13]*

---

**6.** (a) Work out $12+6\div3+1\times5$.

*Answer* ..... [1]

(b) Work out $\frac{7}{9}-\frac{3}{5}$.

*Answer* ..... [1]

*[Nov/2015/P11/Q1]*

---

**7.** A number written as the product of its prime factors is $2^2\times5^2\times7$.

(a) Evaluate this number.

*Answer* ..... [1]

(b) The lowest common multiple of $2^2 \times 5^2 \times 7$ and another number, $N$, is $2^2 \times 3 \times 5^2 \times 7^2$.
Find the lowest possible value of $N$.

*Answer* $N$ = ..... [1]

*[Nov/2015/P11/Q3]*

---

**8.** Here is a list of numbers.

$$-8 \quad -5 \quad -3 \quad -2 \quad 0 \quad 2 \quad 4 \quad 9$$

(a) Write down two numbers from the list that have a difference of 10.

*Answer* ..... and ..... [1]

(b) Find the sum of the numbers in the list.

*Answer* ..... [1]

(c) It is given that $-4 \leq 2x \leq 7$.
Write down all the numbers from the list which satisfy this inequality.

*Answer* ..... [1]

*[Nov/2015/P11/Q10]*

---

**9.** (a) Evaluate $0.03 \times 0.3$.

*Answer* ..... [1]

(b) Evaluate $5 - 2(3 - 1.4)$.

*Answer* ..... [1]

*[Nov/2015/P12/Q1]*

---

**10.** $a$, $b$, $c$, $d$ and $e$ are five numbers, such that

$$d < a < c$$
$$a < e < c$$
$$a < b < e$$

Arrange these numbers in order, starting with the smallest.

*Answer* ..... , ..... , ..... , ..... , ..... [2]
*smallest*

*[Nov/2015/P12/Q8]*

---

**11.** (a) Express 198 as the product of its prime factors.

*Answer* ..... [1]

(b) $M = 2^2 \times 3 \times 5^2 \qquad N = 2^3 \times 3^2 \times 7$

(i) Find the largest number that divides exactly into $M$ and $N$.

*Answer* ..... [1]

(ii) Find the smallest value of $k$, such that $M \times k$ is a cube number.

*Answer* $k =$ ..... [1]

*[Nov/2015/P12/Q12]*

---

**12.** (a) Evaluate $12 - 6 \div 3 + 4$.

*Answer* ..... [1]

(b) Evaluate $0.3 \times 1.5$.

*Answer* ..... [1]

*[June/2016/P11/Q1]*

---

**13.** (a) Evaluate $\frac{2}{3} - \frac{5}{8}$.

*Answer* ..... [1]

(b) Evaluate $\frac{1}{3} \div \frac{7}{9}$, giving your answer as a fraction in its lowest terms.

*Answer* ..... [1]

*[June/2016/P11/Q2]*

---

**14.** (a) Express 500 as the product of its prime factors.

*Answer* ..... [1]

(b) $M = 2 \times 3^2 \qquad N = 2^4 \times 3^2$

Find the values of $p$ and $q$ when

(i) $M \times N = 2^p \times 3^q$,

*Answer* $p$ = ..... $q$ = ..... [1]

(ii) $M \div N = 2^p \times 3^q$,

*Answer* $p$ = ..... $q$ = ..... [1]

(iii) $N^2 = 2^p \times 3^q$.

*Answer* $p$ = ..... $q$ = ..... [1]

*[June/2016/P11/Q21]*

---

**15.** (a) Evaluate $(2.05 + 1.4) \times 0.2$.

*Answer* ..... [1]

(b) Evaluate $1\frac{1}{3} - \frac{4}{5}$.

*Answer* ..... [1]

*[June/2016/P12/Q1]*

---

**16.** Complete the table.

| Fraction | | Decimal | | Percentage |
|---|---|---|---|---|
| $\frac{1}{2}$ | = | 0.5 | = | 50% |
| $\frac{3}{20}$ | = | ..... | = | ..... |
| ..... | = | ..... | = | 62.5% |

[2]

*[June/2016/P12/Q4]*

---

**17.** (a) Express 96 as a product of its prime factors.

*Answer* ..... [1]

(b) 24 is a common factor of 96 and the integer $n$.
Given that $n$ is less than 96, find the largest possible value of $n$.

*Answer* ..... [1]

*[June/2016/P12/Q6]*

---

**18.** Write these values in order of size, starting with the smallest.

$$2^5 \quad 5^2 \quad \sqrt[3]{1000} \quad 27^0$$

*Answer* ....., ....., ....., ..... [1]
*smallest*

*[June/2016/P12/Q13(a)]*

---

**19.** (a) Evaluate $3\frac{1}{6} - 2\frac{3}{5}$.

*Answer* ..... [1]

(b) Evaluate $0.03 \times 0.11$.

*Answer* ..... [1]

*[Nov/2016/P11/Q1]*

---

**20.** (a) Express $32\frac{1}{2}\%$ as a fraction in its simplest form.

*Answer* ..... [1]

(b) Arrange these values in order of size, starting with the smallest.

$$0.38 \qquad \frac{9}{25} \qquad 0.4 \qquad \frac{7}{20}$$

*Answer* ..... , ..... , ..... , ..... [1]
*smallest*

*[Nov/2016/P11/Q3]*

---

# ANSWERS

## Topic 1 - Numbers

**1.** **(a)** $\frac{3}{8}+1\frac{1}{3}=\frac{3}{8}+\frac{4}{3}=\frac{41}{24}$

**(b)** $5-3(2-1.4)=5-1.8=3.2$

**2.** $\frac{4}{5}\quad\frac{3}{4}\quad\frac{31}{40}$

$=0.8\quad 0.75\quad 0.775\quad\Rightarrow\quad\frac{3}{4}\quad\frac{31}{40}\quad\frac{4}{5}$

**3.** **(a)** $\frac{1.3+2.9}{0.2}=\frac{4.2}{0.2}=\frac{42}{2}=21$

**(b)** $2\frac{1}{4}\times\frac{1}{5}=\frac{9}{4}\times\frac{1}{5}=\frac{9}{20}$

**4.** $\frac{13}{20}\quad 0.7\quad\frac{7}{12}\quad 0.64\quad\frac{5}{8}$

$=0.65\quad 0.7\quad 0.583\quad 0.64\quad 0.625$

$\therefore\quad\frac{7}{12},\quad\frac{5}{8},\quad 0.64,\quad\frac{13}{20},\quad 0.7$

**5.** **(a)** $60=2^2\times3\times5$

**(b)** $60m=2^2\times3\times5\times(3\times5).\quad\therefore\ m=15$

**(c)** $60=2\times2\times3\times5,\quad 180=2\times2\times3\times3\times5$

$\therefore$ smallest value of $n=3\times3=9$

**6.** **(a)** $12+6\div3+1\times5=12+2+5=19$

**(b)** $\frac{7}{9}-\frac{3}{5}=\frac{8}{45}$

**7.** **(a)** $2^2\times5^2\times7=4\times25\times7=700$

**(b)** $700=2^2\times5^2\times7.\quad\text{LCM}=2^2\times3\times5^2\times7^2.$

$\therefore$ Lowest value of $N=3\times7^2=147$

**8.** **(a)** $2$ and $-8$.

**(b)** $-8-5-3-2+0+2+4+9=-3$

**(c)** $-4\le2x\le7\quad\Rightarrow\quad-2\le x\le3.5$

$\therefore$ Numbers are, $-2, 0, 2$

**9.** **(a)** $0.03\times0.3=0.009$

**(b)** $5-2(3-1.4)=5-2(1.6)=1.8$

**10.** $d<a<b<e<c$

**11.** **(a)** $198=2\times3^2\times11$

**(b)** **(i)** GCD of $M$ and $N=2^2\times3=12$

**(ii)** $M\times k=2^2\times3\times5^2\times(2\times3^2\times5)$

$\therefore\ k=2\times3^2\times5=90$

**12.** **(a)** $12-6\div3+4=12-2+4=14$

**(b)** $0.3\times1.5=0.45$

**13.** **(a)** $\frac{2}{3}-\frac{5}{8}=\frac{1}{24}$

**(b)** $\frac{1}{3}\div\frac{7}{9}=\frac{1}{3}\times\frac{9}{7}=\frac{3}{7}$

**14.** **(a)** $500=2\times2\times5\times5\times5=2^2\times5^3$

**(b)** **(i)** $M\times N=2^5\times3^4.\quad\therefore\ p=5,\quad q=4$

**(ii)** $M\div N=\frac{2\times3^2}{2^4\times3^2}=2^{-3}\times3^0,$

$\therefore\ p=-3,\quad q=0$

**(iii)** $N^2=2^8\times3^4.\quad\therefore\ p=8,\quad q=4$

**15.** **(a)** $(2.05+1.4)\times0.2=3.45\times0.2=0.69$

**(b)** $1\frac{1}{3}-\frac{4}{5}=\frac{4}{3}-\frac{4}{5}=\frac{20-12}{15}=\frac{8}{15}$

**16.**

| Fraction | | Decimal | | Percentage |
|---|---|---|---|---|
| $\frac{1}{2}$ | = | 0.5 | = | 50% |
| $\frac{3}{20}$ | = | **0.15** | = | **15%** |
| **$\frac{5}{8}$** | = | **0.625** | = | 62.5% |

**17.** **(a)** $96=2\times2\times2\times2\times2\times3=2^5\times3$

**(b)** $n$ is a multiple of 24. Thus, the largest value of $n$ less than $96=24\times3=72$

**18.** $2^5 \quad 5^2 \quad \sqrt[3]{1000} \quad 27^0$

$= 32 \quad 25 \quad 10 \quad 1$

$\therefore \quad 27^0, \quad \sqrt[3]{1000}, \quad 5^2, \quad 2^5$

**19. (a)** $3\frac{1}{6} - 2\frac{3}{5} = \frac{19}{6} - \frac{13}{5} = \frac{17}{30}$

**(b)** $3 \times 11 = 33$

$\therefore \; 0.03 \times 0.11 = 0.0033$

**20. (a)** $32\frac{1}{2}\% = \frac{65}{2} \times \frac{1}{100} = \frac{13}{40}$

**(b)** $0.38 \quad \frac{9}{25} \quad 0.4 \quad \frac{7}{20}$

$= 0.38 \quad 0.36 \quad 0.4 \quad 0.35$

$\therefore \quad \frac{7}{20} \quad \frac{9}{25} \quad 0.38 \quad 0.4$

**21. (a)** $9.03 - (4.273 + 2.3) = 9.03 - 6.573 = 2.457$

**(b)** $\frac{8}{9} - \frac{6}{7} = \frac{56-54}{63} = \frac{2}{63}$

**22. (a)** $0.192 \times 643 = \frac{192}{1000} \times (64.3 \times 10)$

$= \frac{12345.6}{100} = 123.456$

**(b)** $\frac{12.3456}{192} = \frac{12345.6}{192} \times \frac{1}{1000}$

$= 64.3 \times \frac{1}{1000} = 0.0643$

**23.** $74\% \quad -0.7 \quad 0.7 \quad -\frac{3}{4}$

$= 0.74 \quad -0.7 \quad 0.7 \quad -0.75$

$\therefore \quad -\frac{3}{4} \quad -0.7, \quad 0.7, \quad 74\%$

**24. (a)** $0.2 \times 0.08 = 0.016$

**(b)** $2 \times (3 + 4) \times 5 = 70$

**25. (a)** $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

**(b)** 2 and 13

**26. (a)** $\frac{3}{5} - \frac{1}{8} = \frac{24-5}{40} = \frac{19}{40}$

**(b)** $A \times \frac{3}{7} = \frac{2}{5} \quad \Rightarrow \quad A = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$

**(c)** $\left(\frac{5}{8} + \frac{2}{3}\right) \times \frac{1}{2} = \frac{31}{24} \times \frac{1}{2} = \frac{31}{48}$

**27. (a)** $\frac{4}{5} - \frac{1}{3} = \frac{12-5}{15} = \frac{7}{15}$

**(b)** $2 \times 6 = 12. \quad \therefore \; 0.2 \times 0.006 = 0.0012$

**28. (i)** $54 = 2 \times 3 \times 3 \times 3$

**(ii)** $54m = 2 \times 3^3 \times (2^2). \quad \therefore \; m = 4$

**29. (a)** $1\frac{3}{8} - \frac{2}{3} = \frac{11}{8} - \frac{2}{3} = \frac{17}{24}$

**(b)** $0.4 \times 1.3 = 0.52$

**30. (a)** $\frac{6}{7} - \frac{3}{5} = \frac{30-21}{35} = \frac{9}{35}$

**(b)** $\frac{90}{0.45} \times \frac{100}{100} = \frac{90 \times 100}{45} = 200$

**31. (a)** $\frac{3.5 - 1.9}{0.8} = \frac{1.6}{0.8} = \frac{16}{8} = 2$

**(b)** $9 + (6 \div 3) - 4 = 9 + 2 - 4 = 7$

**32. (a)** $\frac{15}{100} \times 80 = 12$ **(b)** $\frac{3}{5} - \frac{2}{7} = \frac{11}{35}$

**33.** $\frac{1}{3} \quad 0.32 \quad \frac{15}{40} \quad 0.3 \quad \frac{9}{31}$

$= 0.333 \quad 0.32 \quad 0.375 \quad 0.3 \quad 0.29$

$\therefore \quad \frac{9}{31} \quad 0.3 \quad 0.32 \quad \frac{1}{3} \quad \frac{15}{40}$

**34. (a)** $\frac{4}{11} - \frac{2}{7} = \frac{28-22}{77} = \frac{6}{77}$

**(b)** $0.9 \times 0.011 = \frac{9}{10} \times \frac{11}{1000} = 0.0099$

**35. (a)** $-1.2 \quad -0.3 \quad 0.05 \quad 0.2 \quad 1.3$

**(b) (i)** $\frac{-1.2 - 0.3 + 0.05 + 0.2 + 1.3}{5} = 0.01$

**(ii)** Range, $1.3 - (-1.2) = 2.5$

**36. (a)** $\frac{2}{7} + \frac{1}{5} = \frac{17}{35}$ **(b)** $\frac{2}{5} \times 1\frac{1}{5} = \frac{2}{5} \times \frac{6}{5} = \frac{12}{25}$

**37. (a)** $17\frac{1}{2}\% = \frac{35}{2} \times \frac{1}{100} = \frac{7}{40}$

**(b)** $6 + 4(1 - 0.4) = 6 + 2.4 = 8.4$

**38. (a)** $\sqrt{19}$ **(b)** Final digit = 4

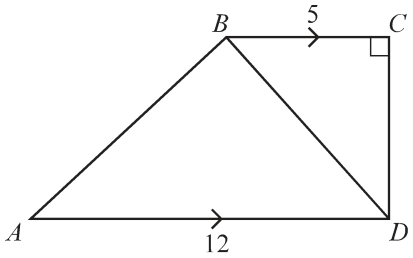
**39. (a)** $1200 = 2^4 \times 3 \times 5^2$

**(b)** $120n = 2^3 \times 3 \times 5 \times (2 \times 3 \times 5). \quad \therefore \; n = 30$

# TOPIC 28

# Mensuration

**1.** *ABCD* is a quadrilateral with *BC* parallel to *AD*.
*CD* is perpendicular to *BC*. *BC* = 5 cm and *AD* = 12 cm.
The area of triangle *BCD* is 20 cm$^2$.




(a) Find *CD*.

*Answer* ..... cm [1]

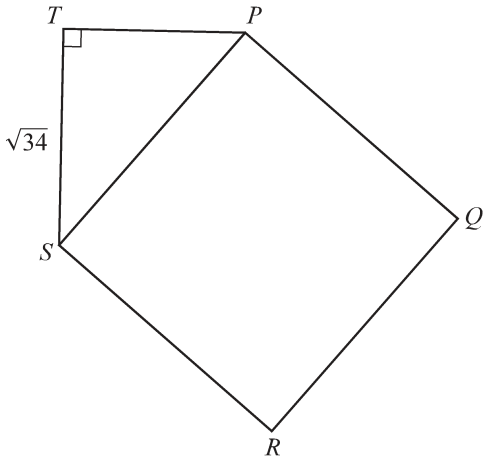
(b) Find the area of triangle *ABD*.

*Answer* ..... cm$^2$ [1]

*[Nov/2015/P11/Q2]*

---

**2.** The diagram shows a square *PQRS* and a right-angled triangle *PST*. The area of the square is 50 cm$^2$.
$ST = \sqrt{34}$ cm. Calculate *PT*.




*Answer* ..... cm [2]

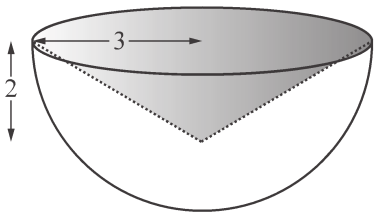
*[Nov/2015/P11/Q5]*

---

**3.** **[The volume of a sphere is $\frac{4}{3}\pi r^3$][The volume of a cone is $\frac{1}{3}\pi r^2 h$]**

A cone is removed from a solid wooden hemisphere of radius 3 cm.
The cone has radius 3 cm and height 2 cm.

The volume of wood remaining is $k\pi$ cm$^3$.
Find *k*.

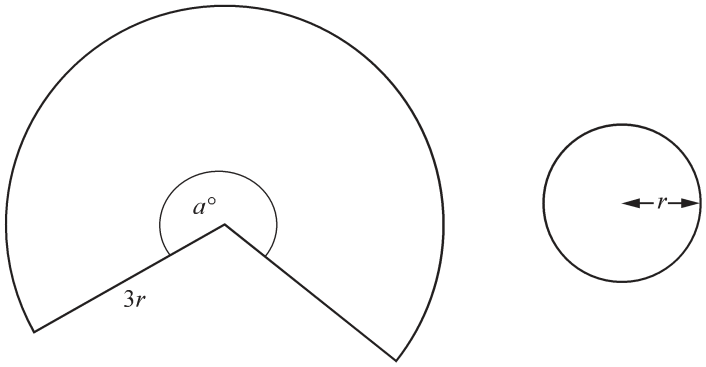



*Answer* *k* = ..... [3]

*[June/2016/P11/Q14]*

---

**4.**




The diagram shows a sector of a circle with radius $3r$ cm and angle $a°$ and a circle with radius $r$ cm.
The ratio of the area of the sector to the area of the circle with radius $r$ cm is 8 : 1.

(a) Find the value of $a$.

*Answer* $a$ = ..... [3]

(b) Find an expression, in terms of $\pi$ and $r$, for the perimeter of the sector.

*Answer* ..... cm [2]

*[June/2016/P12/Q24]*

---

**5. [The volume of a sphere is $\frac{4}{3}\pi r^3$]**

During a storm, raindrops fall into a cylinder which stands on horizontal ground.
The cylinder was empty before the storm started.

The cylinder has radius 20 mm.
Each raindrop is a sphere of radius 2 mm.
After the storm, the depth of water in the cylinder is 16 mm.

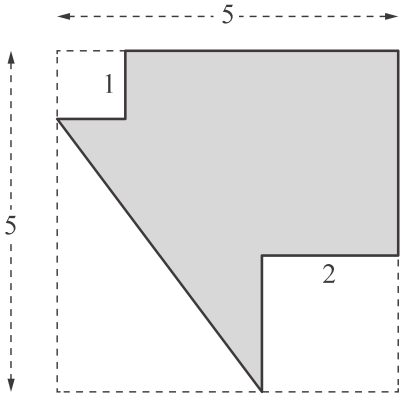
Calculate the number of raindrops that fell into the cylinder.

*Answer* ..... [3]

*[Nov/2016/P12/Q21]*

---

**6.** The diagram shows a square piece of card, from which a triangle and two small squares are removed. All lengths on the diagram are in centimetres.




(a) Calculate the area of the shaded card.

*Answer* ..... $cm^2$ [2]

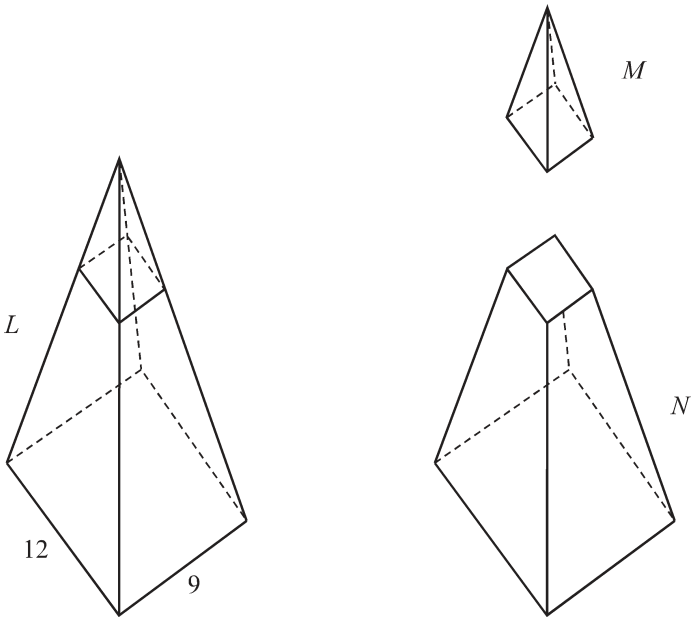
(b) Calculate the perimeter of the shaded card.

*Answer* ..... cm [2]

*[Nov/2016/P12/Q23]*

---

**7.** **[ Volume of a pyramid = $\frac{1}{3}$ × base area × perpendicular height ]**




The diagrams show a solid pyramid *L* cut into two parts, *M* and *N*, by a plane parallel to its base.
The base of pyramid *L* is a rectangle 9 cm by 12 cm.
The perpendicular height of pyramid *L* is 30 cm.

(a) Work out the volume of pyramid *L*.

*Answer* ..... $cm^3$ [1]

(b) The perpendicular height of pyramid $M$ is $\frac{1}{3}$ of the perpendicular height of pyramid $L$.

(i) Express the volume of $M$ as a fraction of the volume of $L$.

*Answer* ..... [1]

(ii) Calculate the volume of the solid $N$.

*Answer* ..... $cm^3$ [2]

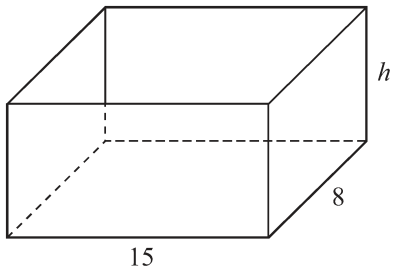
*[June/2017/P11/Q15]*

---

**8.** A container is made out of thin material in the shape of a cuboid with an open top.

The container has length 15 cm and width 8 cm.

The volume of the container is 720 $cm^3$.




(a) Calculate the height, $h$ cm, of the container.

*Answer* ..... cm [2]

(b) Calculate the surface area of the **outside** of the container.

*Answer* ..... $cm^2$ [2]

(c) Liquid is poured into the container.

The liquid fills 60% of the container.

Calculate the height of the liquid in the container.

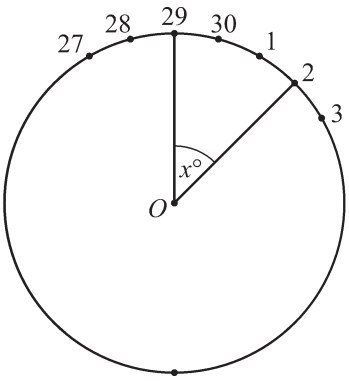
*Answer* ..... cm [1]

*[June/2017/P12/Q22]*

**9.** The diagram represents a vertical, circular fairground wheel which turns about its centre $O$.
The wheel has 30 seats, equally spaced around the circumference, numbered consecutively from 1 to 30.

The diagram, which is not drawn to scale, shows seven of the seats, labelled with the seat number.

The seat number 29 is at the top of the wheel.




(a) What is the number of the seat which is at the bottom of the wheel?

*Answer* ..... [1]

(b) Calculate the angle $x°$, as shown on the diagram.

*Answer* ..... [1]

(c) Work out the angle of elevation of seat 2 from the bottom of the wheel.

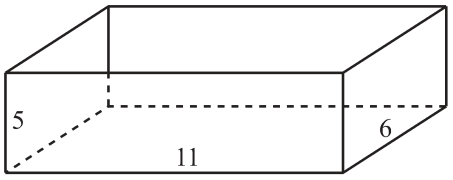
*Answer* ..... [2]

*[Nov/2017/P11/Q18]*

---

**10.** An **open** rectangular tray has inside measurements

length 11 cm width 6 cm height 5 cm.




(a) Calculate the total surface area of the four sides and base of the inside of the tray.

*Answer* ..... $cm^2$ [2]

(b) Cubes are placed in the tray and a lid is placed on top.
Each cube has an edge of 2 cm.

Find the maximum number of cubes that can be placed in the tray.

*Answer* ..... [1]

*[Nov/2017/P12/Q18]*

---

# ANSWERS

## Topic 28 - Mensuration

**1.** **(a)** Area of $\Delta BCD = \frac{1}{2}(5)(CD)$

$\Rightarrow\ 20 = \frac{1}{2}(5)(CD) \quad \Rightarrow \quad CD = 8$ cm

**(b)** Area of $ABD = \frac{1}{2}(12)(8) = 48$ cm$^2$

**2.** Area of square, $l^2 = 50 \quad \Rightarrow \quad l = \sqrt{50}$

Using Pythagoras on $\Delta PST$,

$PT^2 = (\sqrt{50})^2 - (\sqrt{34})^2 \quad \Rightarrow \quad PT = 4$ cm

**3.** Vol. of wood = Vol.$_{\text{hemisphere}}$ − Vol.$_{\text{cone}}$

$\Rightarrow\ k\pi = \frac{1}{2}\left(\frac{4}{3}\pi(3)^3\right) - \left(\frac{1}{3}\pi(3)^2(2)\right)$

$\Rightarrow\ k\pi = 18\pi - 6\pi \quad \Rightarrow \quad k = 12$

**4.** **(a)** $\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{8}{1}$

$\Rightarrow\ \frac{\frac{a^\circ}{360} \times \pi(3r)^2}{\pi r^2} = \frac{8}{1} \quad \Rightarrow \quad a^\circ = 320^\circ$

**(b)** Perimeter = length of arc $+ 3r + 3r$

$= \frac{320^\circ}{360^\circ} \times 2\pi(3r) + 6r = \frac{16\pi r}{3} + 6r$

**5.** Vol. of water in the cylinder $= \pi(20)^2(16)$

$= 6400\pi$ mm$^3$

Vol. of one raindrop $= \frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi$ mm$^3$

$\therefore$ Number of raindrops $= \frac{6400\pi}{\frac{32}{3}\pi} = 600$

**6.** **(a)** Shaded area = area of larger square − (area of 2 small squares + area of $\Delta$)

$= (5)^2 - \left((1)^2 + (2)^2 + \frac{1}{2}(3)(4)\right) = 14$ cm$^2$

**(b)** By Pythagoras, slant ht. of $\Delta = \sqrt{3^2 + 4^2}$

$= 5$ cm

Perimeter $= 5 + 2 + 2 + 3 + 4 + 1 + 1 = 18$ cm

**7.** **(a)** Volume of $L = \frac{1}{3} \times (9 \times 12) \times 30 = 1080$ cm$^3$

**(b)** **(i)** Height of $M = \frac{1}{3} \times 30 = 10$ cm

$\therefore\ \frac{\text{Volume of } M}{\text{Volume of } L} = \left(\frac{10}{30}\right)^3 = \frac{1}{27}$

**(ii)** Vol. of $M = \frac{1}{27} \times$ Vol. of $L$

$= \frac{1}{27} \times 1080 = 40$ cm$^3$

$\therefore$ Vol. of $N = 1080 - 40 = 1040$ cm$^3$

**8.** **(a)** Vol. of cuboid $= 15 \times 8 \times h$

$\Rightarrow\ 720 = 15 \times 8 \times h \quad \Rightarrow \quad h = 6$ cm.

**(b)** Surface area $= (15 \times 8) + 2(15 \times 6) + 2(8 \times 6)$

$= 396$ cm$^2$

**(c)** Volume of liquid $= \frac{60}{100} \times 720 = 432$ cm$^3$

$\Rightarrow\ 15 \times 8 \times h = 432 \quad \Rightarrow \quad h = 3.6$ cm

**9.** **(a)** Seat number 14.

**(b)** The sector has 3 spaces between seats.

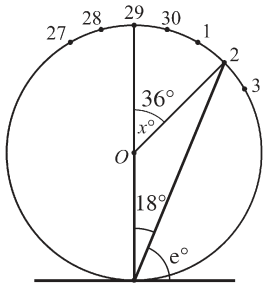
So there are $\frac{30}{3} = 10$ sectors in the circle.

$\therefore\ x^\circ = \frac{360^\circ}{10} = 36^\circ$

**(c)** From figure,

angle of elevation

$= 90^\circ - 18^\circ = 72^\circ$




**10.** **(a)** Surface area $= (11 \times 6) + 2(11 \times 5) + 2(6 \times 5)$

$= 236$ cm$^2$

**(b)** Number of cubes that fill the length $= 5$

Number of cubes that fill the width $= 3$

Number of cubes that fill the height $= 2$

$\therefore$ Total No. of cubes $= 5 \times 3 \times 2 = 30$

**11.** By Pythagoras, $BC = \sqrt{6^2 + 7^2} = \sqrt{85}$

Req. Area $= (\sqrt{85})^2 + \frac{1}{2}(6)(7) = 106 \text{ cm}^2$

**12.** **(a)** $20 \times 40 \times h = 2400 \quad \Rightarrow \quad h = 3 \text{ cm}$

**(b)** Volume $= 1000 \times 2400$

$= 2400000 \text{ cm}^3 = \frac{2400000}{1000000} = 2.4 \text{ m}^3$

**(c)** $\frac{\text{Vol. of bigger slab}}{2400} = \left(\frac{60}{40}\right)^3$

Vol. of bigger slab $= \frac{27}{8} \times 2400 = 8100 \text{ cm}^3$

**13.** Length of base $= y$ cm, height $= 2y$ cm

Total surface area $= 2(y^2) + 4(y \times 2y)$

$\Rightarrow \quad 360 = 10y^2 \quad \Rightarrow \quad y = 6 \text{ cm}$

$\therefore$ height of the cuboid $= 2(6) = 12$ cm.

**14.** Shaded area of sector $AOB = \frac{60}{360}\pi 3^2 = \frac{3}{2}\pi$

Empty area of $AOB = \frac{60}{360}(\pi)(6^2 - 3^2) = \frac{9}{2}\pi$

Total area of ring $= \pi(6^2 - 3^2) = 27\pi$

$\therefore$ Total shaded area $= 27\pi + \frac{3}{2}\pi - \frac{9}{2}\pi$

$\Rightarrow \quad k\pi = 27\pi + \frac{3}{2}\pi - \frac{9}{2}\pi \quad \Rightarrow \quad k = 24$

**15.** Total vol. of 400 drops $= 0.08 \times 400 = 32 \text{ cm}^3$

Volume of cuboid $= 5 \times 4 \times h$

$\Rightarrow \quad 32 = 20h \quad \Rightarrow \quad h = 1.6 \text{ cm}$

$\therefore$ change in water level $= 1.6$ cm.

**16.** $6^2 = l \times 4 \quad \Rightarrow \quad l = 9$ cm. Also, $TU = 3$ cm

Perimeter $= 6 + 4 + 9 + 4 + 3 + 6 + 6 = 38$ cm.

**17.** $\pi(28)(y) = 2\pi(15)(42)$

$\Rightarrow \quad y = \frac{2(15)(42)}{28} = 45$ cm.

**18.** Area of trapezium $= 85 \text{ cm}^2$

$\Rightarrow \quad \frac{1}{2}(h)(7 + 10) = 85$

$\Rightarrow \quad \frac{17}{2}h = 85 \quad \Rightarrow \quad h = 10$ cm.

**19.** **(a)** Let $l$ be the length of edge of 2nd cube.

Total volume of 2 cubes $= 5^3 + l^3$

$\Rightarrow \quad 152 = 5^3 + l^3$

$\Rightarrow \quad 152 = 125 + l^3$

$\Rightarrow \quad l^3 = 27 \quad \Rightarrow \quad l = 3 \text{ cm}$

**(b)** A cube has 12 edges

$\therefore$ Total length of all edges $= 5 \times 12 = 60$ cm

**20.** $85 \text{ mm}^2 = \frac{85}{100} \text{ cm}^2 = 0.85 \text{ cm}^2$

$\therefore$ Shaded area $= 9 - 0.85 = 8.15 \text{ cm}^2$

**21.** Area of sector $= \frac{360° - 80°}{360°} \times \pi(3)^2$

$= \frac{280°}{360°} \times \pi(9) = 7\pi \text{ cm}^2$

**22.** **(a)** Slant height of cone is,

$l = y + \frac{25}{100}(y) = \frac{125}{100}y = \frac{5}{4}y$ cm

Given that,

Surface area of sphere

= total surface area of cone.

$\Rightarrow \quad 4\pi R^2 = \pi y^2 + \pi(y)\left(\frac{5}{4}y\right)$

$\Rightarrow \quad 4\pi R^2 = \pi y^2 + \frac{5}{4}\pi y^2$

$\Rightarrow \quad 4\pi R^2 = \frac{9}{4}\pi y^2$

$\Rightarrow \quad 16R^2 = 9y^2$

$\Rightarrow \quad y^2 = \frac{16R^2}{9} \quad \Rightarrow \quad y = \frac{4R}{3}$

**(b)** By Pythagoras, height of cone is,

$h = \sqrt{\left(\frac{5}{4}y\right)^2 - y^2} = \sqrt{\frac{9}{16}y^2} = \frac{3}{4}y$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi y^2\left(\frac{3}{4}y\right)$

$= \frac{1}{4}\pi y^3 \quad \left(\text{Subst. } y = \frac{4R}{3} \text{ from (a)}\right)$

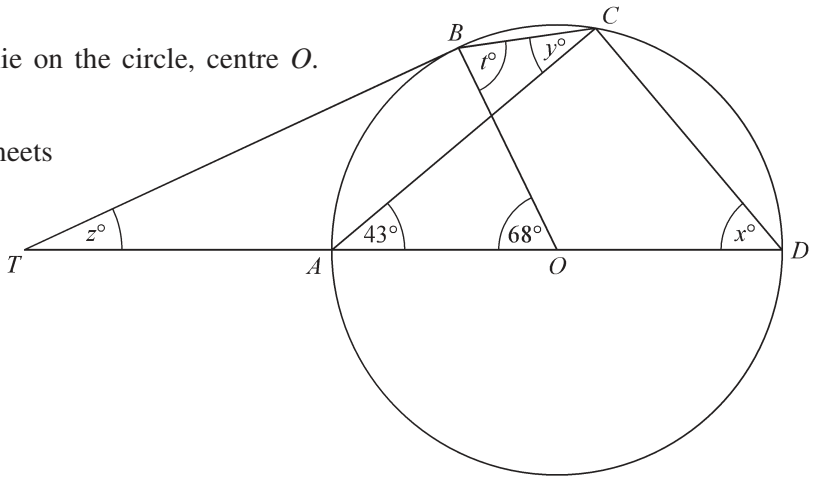
$= \frac{1}{4}\pi\left(\frac{4R}{3}\right)^3$

$= \frac{1}{4}\pi\left(\frac{64R^3}{27}\right) = \frac{16\pi R^3}{27} \text{ cm}^3$

# TOPIC 27

# Circle Properties

**1.** In the diagram, $A$, $B$, $C$ and $D$ lie on the circle, centre $O$.
$AD$ is a diameter.
The tangent to the circle at $B$ meets the line $DA$ produced at $T$.
$A\hat{O}B = 68°$ and $C\hat{A}O = 43°$.




(a) Find $x$.

*Answer* $x$ = ..... [1]

(b) Find $y$.

*Answer* $y$ = ..... [1]

(c) Find $z$.

*Answer* $z$ = ..... [1]

(d) Find $t$.

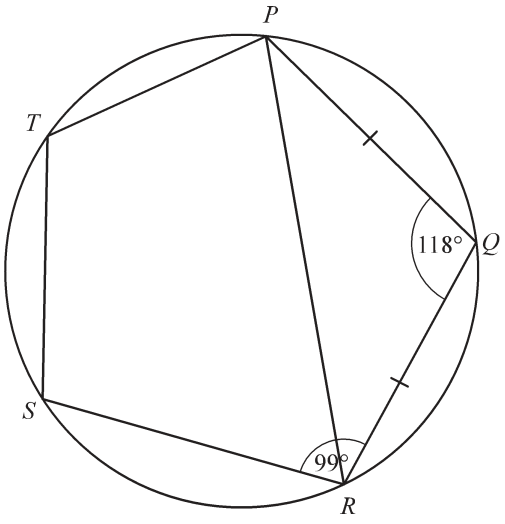
*Answer* $t$ = ..... [1]

*[June/2015/P11/Q19]*

---

**2.** (a) $P$, $Q$, $R$, $S$ and $T$ are points on the circumference of a circle.
$PQ = QR$.
$P\hat{Q}R = 118°$ and $Q\hat{R}S = 99°$.

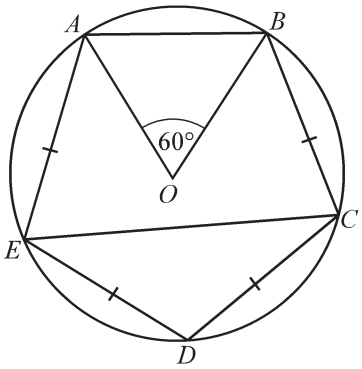
Find $P\hat{T}S$.
Show all your working.




*Answer* $P\hat{T}S$ = ..... [2]

(b) *A*, *B*, *C*, *D* and *E* are points on the circumference of a circle, centre *O*.

$AE = ED = DC = CB$ and $A\hat{O}B = 60°$.




(i) Find $E\hat{C}D$.
Show all your working.

*Answer* $E\hat{C}D =$ ..... [2]

(ii) The radius of the circle is 12 cm.
Calculate the length of the minor arc *AB*.
Use $\pi = 3.14$.

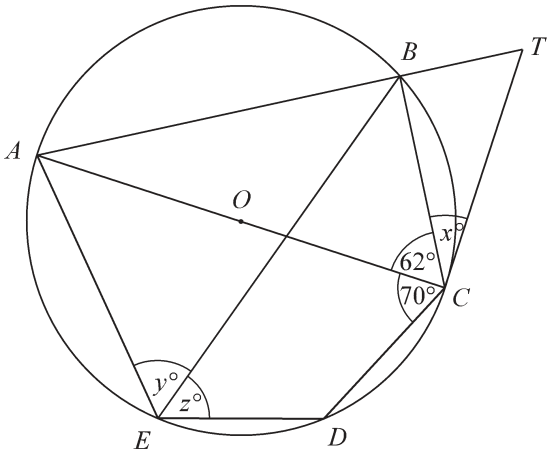
*Answer* ..... cm [2]

*[Nov/2015/P11/Q24]*

---

**3.** In the diagram, *A*, *B*, *C*, *D* and *E* lie on the circle, centre *O*.
*AC* is a diameter.
The tangent to the circle at *C* meets the line *AB* produced at *T*.

$A\hat{C}B = 62°$ and $A\hat{C}D = 70°$.




(a) Find *x*.

*Answer* $x =$ ..... [1]

(b) Find *y*.

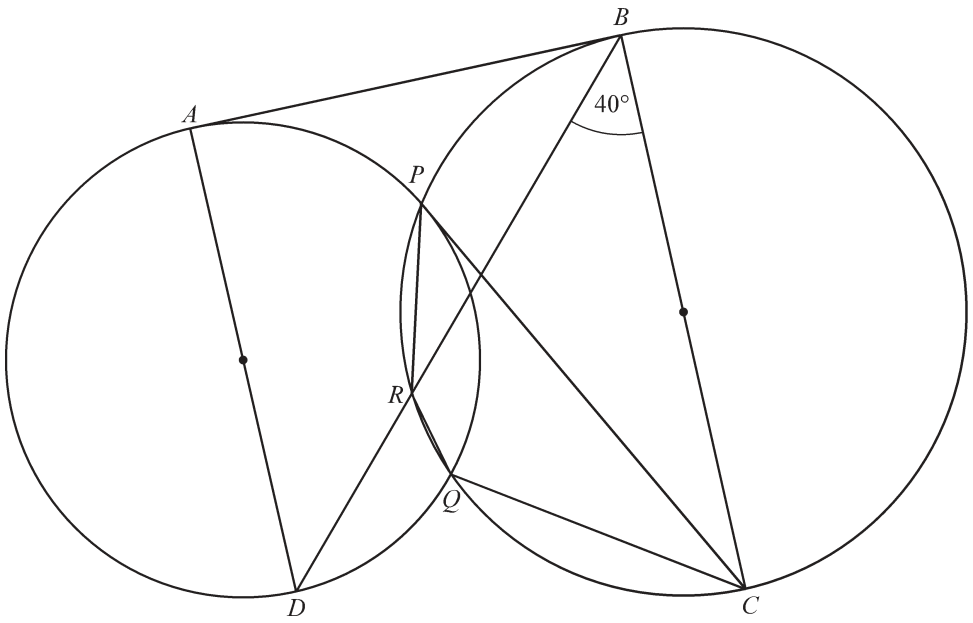
*Answer* $y =$ ..... [1]

(c) Find *z*.

*Answer* $z =$ ..... [1]

*[Nov/2015/P12/Q17]*

**4.**




In the diagram, the two circles intersect at $P$ and $Q$.
The line $AB$ is a tangent to the circles at $A$ and $B$.
$AD$ and $BC$ are diameters.
$BD$ intersects the larger circle at $R$.

$D\hat{B}C = 40°$.

(a) Find $C\hat{P}R$.

*Answer* $C\hat{P}R =$ ..... [1]

(b) Find $C\hat{Q}R$.

*Answer* $C\hat{Q}R =$ ..... [1]

(c) Find $A\hat{B}D$.

*Answer* $A\hat{B}D =$ ..... [1]

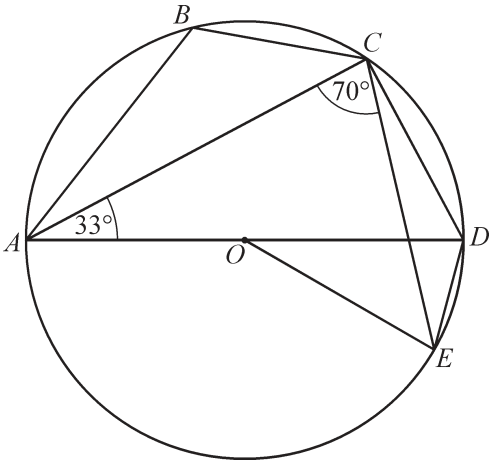
(d) Find $A\hat{D}B$.

*Answer* $A\hat{D}B =$ ..... [1]

*[June/2016/P11/Q19]*

**5.** In the diagram, the points *A*, *B*, *C*, *D* and *E* lie on the circle centre *O*.
*AD* is a diameter.

$D\hat{A}C = 33°$ and $A\hat{C}E = 70°$.




(a) Find $C\hat{D}A$.

*Answer* $C\hat{D}A =$ ..... [1]

(b) Find $D\hat{E}C$.

*Answer* $D\hat{E}C =$ ..... [1]

(c) Find $A\hat{B}C$.

*Answer* $A\hat{B}C =$ ..... [1]

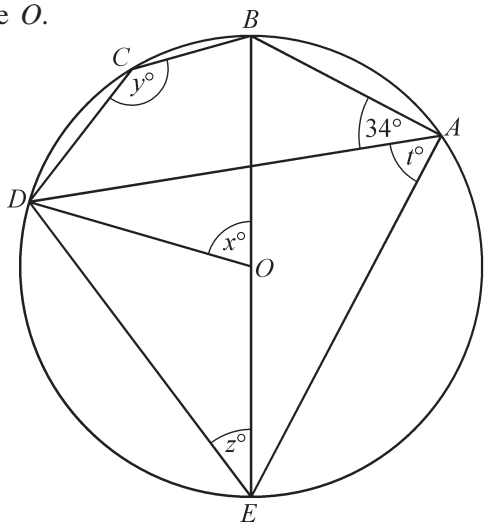
(d) Find reflex $E\hat{O}A$.

*Answer* reflex $E\hat{O}A =$ ..... [1]

*[Nov/2016/P11/Q17]*

---

**6.** In the diagram, *A*, *B*, *C*, *D* and *E* lie on the circle, centre *O*.
*BOE* is a straight line.
$D\hat{A}B = 34°$.




(a) Find *x*.

*Answer* $x =$ ..... [1]

(b) Find *y*.

*Answer* $y =$ ..... [1]

(c) Find *z*.

*Answer* $z =$ ..... [1]

(d) Find *t*.

*Answer* $t =$ ..... [1]

*[Nov/2016/P12/Q24]*

# ANSWERS

## Topic 27 - Circle Properties

**1.** **(a)** $\Delta ACD$ is a right angled $\Delta$,
$\therefore\ x^\circ = 90^\circ - 43^\circ = 47^\circ$

**(b)** $y^\circ = \frac{1}{2}(68^\circ) = 34^\circ$

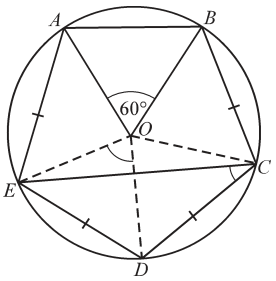
**(c)** $\Delta OBT$ is a right angled $\Delta$,
$\therefore\ z^\circ = 90^\circ - 68^\circ = 22^\circ$

**(d)** $B\hat{O}D = 180^\circ - 68^\circ = 112^\circ$,
$B\hat{C}D = 90^\circ + 34^\circ = 124^\circ$
In quadrilateral $OBCD$
$t^\circ + 124^\circ + 47^\circ + 112^\circ = 360^\circ \Rightarrow t^\circ = 77^\circ$

**2.** **(a)** In $\Delta PQR$, $Q\hat{R}P = \frac{180^\circ - 118^\circ}{2} = 31^\circ$
$\therefore\ P\hat{R}S = 99^\circ - 31^\circ = 68^\circ$
$PRST$ is a cyclic quadrilateral,
$\therefore\ P\hat{T}S = 180^\circ - 68^\circ = 112^\circ$

**(b)** **(i)** From figure,

$E\hat{O}D = \frac{360^\circ - 60^\circ}{4}$
$= 75^\circ$
$\therefore\ E\hat{C}D = \frac{1}{2}(75^\circ)$
$= 37.5^\circ$




**(ii)** Arc length $AB = \frac{60^\circ}{360^\circ} \times 2 \times \pi \times 12$
$= 12.56$ cm.

**3.** **(a)** $A\hat{C}T = 90^\circ$, $\therefore\ x^\circ = 90^\circ - 62^\circ = 28^\circ$

**(b)** $y^\circ = 62^\circ$ ($\angle$s in the same segment)

**(c)** $BCDE$ is a cyclic quadrilateral,
$z^\circ + 70^\circ + 62^\circ = 180^\circ \Rightarrow z^\circ = 48^\circ$

**4.** **(a)** $C\hat{P}R = 40^\circ$ (same segment)

**(b)** $BCQR$ is a cyclic quadrilateral,
$\therefore\ C\hat{Q}R = 180^\circ - 40 = 140^\circ$

**(c)** $A\hat{B}C = 90^\circ$, $\therefore\ A\hat{B}D = 90^\circ - 40^\circ = 50^\circ$

**(d)** $B\hat{A}D = 90^\circ$, $\therefore\ A\hat{D}B = 90^\circ - 50^\circ = 40^\circ$

**5.** **(a)** $\Delta ACD$ is a right angled $\Delta$,
$\therefore\ C\hat{D}A = 90^\circ - 33^\circ = 57^\circ$

**(b)** $D\hat{E}C = 33^\circ$ (same segment)

**(c)** $ABCD$ is a cyclic quadrilateral,
$\therefore\ A\hat{B}C = 180^\circ - 57^\circ = 123^\circ$

**(d)** $A\hat{O}E = 2(70^\circ) = 140^\circ$ (same segment)
$\therefore$ reflex $E\hat{O}A = 360^\circ - 140^\circ = 220^\circ$

**6.** **(a)** $x^\circ = 2 \times 34^\circ = 68^\circ$

**(b)** $ABCD$ is a cyclic quadrilateral,
$\therefore\ y^\circ = 180^\circ - 34^\circ = 146^\circ$

**(c)** $z^\circ = 34^\circ$ ($\angle$s in the same segment).

**(d)** $B\hat{A}E = 90^\circ$, $\therefore\ t^\circ = 90^\circ - 34^\circ = 56^\circ$

**7.** **(a)** $P\hat{T}Q = 35^\circ$ because $P\hat{S}Q = 35^\circ$.
$P\hat{T}Q$ and $P\hat{S}Q$ are in the same segment.

**(b)** $Q\hat{P}T = 90^\circ$, $\therefore\ P\hat{Q}T = 90^\circ - 35^\circ = 55^\circ$

**(c)** In $\Delta PXT$, $T\hat{P}X = 180^\circ - 35^\circ - 125^\circ = 20^\circ$
$\therefore\ S\hat{P}Q = 90^\circ - 20^\circ = 70^\circ$

**(d)** $PQRS$ is a cyclic quadrilateral
$\therefore\ S\hat{R}Q = 180^\circ - 70^\circ = 110^\circ$

**8.** **(a)** $B\hat{C}A = 90^\circ - 38^\circ = 52^\circ$

**(b)** $C\hat{B}T = 90^\circ$, $A\hat{B}T = 90^\circ - 38^\circ = 52^\circ$
$\Delta ABT$ is an isosceles $\Delta$,
$\therefore\ A\hat{T}B = 180^\circ - 2(52^\circ) = 76^\circ$

**9.** **(a)** $x^\circ = 2 \times 53^\circ = 106^\circ$

**(b)** $ABCD$ is a cyclic quadrilateral,
$y^\circ = 180^\circ - 53^\circ = 127^\circ$

**(c)** $ATB$ is an isosceles triangle.
$z^\circ + z^\circ + 62^\circ = 180^\circ \Rightarrow z^\circ = 59^\circ$

**(d)** $\angle OBT = 90^\circ$ (radius $\perp$ tangent)
$\therefore\ t^\circ = 90^\circ - z^\circ = 31^\circ$.