REDSPOT

O LEVEL

CLASSIFIED
WORKED SOLUTIONS

MATHEMATICS
(Paper 2 - All Variants)
(Syllabus 4024)

period	2015 to 2024
contents	June & November, Paper 2 (P21 & P22) Worked Solutions
form	Topic By Topic
compiled for	O Levels

Appointed Agents & Wholesalers in PAKISTAN:

BOOK LAND
Urdu Bazaar, Lahore. Tel: 042-37124656

NATIONAL BOOK SERVICE
Urdu Bazaar, Lahore. Tel: 042-37247310.

LAROSH BOOKS
Urdu Bazaar Lahore. Tel: 042-37312126.

BURHANI BOOK CENTRE
New Urdu Bazar, Karachi, Tel: 021-32634718

MARYAM ACADEMY
Urdu Bazaar, Karachi, Tel: 0331-2425264

TARIQ BOOK TOWN
Samar Garden, Hydari North nazimzbad, Karachi. Tel: 021-34555918, 36649425

REHMAN BOOKS
College Road, Rawalpindi
Tel: 051-5770603, 0313-5618976

WELCOME BOOKS
Soneri Masjid Road, Peshawar Cantt.
Tel: 091-5274932, 0300-5860068

© REDSPOT PUBLISHING

Tel No : 042-35201010
Mobile No : 0300-8447654
E-Mail : info@redspot.com.pk
Website : www.redspot.pk
Address : P.O. Box 5041, Model Town, Lahore, Pakistan.

All rights reserved. No part of this publication may be reproduced, copied or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without permission in writing from the publisher/ distributor.

CONTENTS

CONTENTS

TOPIC 9

Solutions of Equations

1. (a) Expand the brackets and simplify $(x-1)(x^2+x+1)$.

Answer .... [2]

(b) Solve the equation $\frac{3x}{x+2}-\frac{4}{x-2}=3$.

Answer .... [3]

(c) Solve these simultaneous equations.

$$4x-3y=4$$
$$4y-3x=-6.5$$

Answer x = ...

y = ... [4]

[June/2015/P21/Q6]

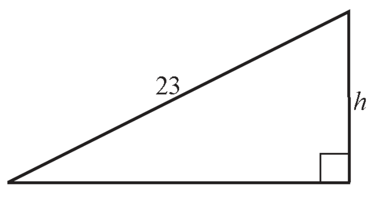
---

2. (a) Solve $\frac{7}{3-2m}=4$.

Answer .... [2]

(b) A right-angled triangle has a base that is 7 cm longer than its height, h cm. The hypotenuse of the triangle is 23 cm.

(i) Show that h satisfies the equation $h^2+7h-240=0$.



[2]

(ii) Write down an expression, in terms of h, for the area of the triangle.

Answer .... cm^2 [1]

(iii) Hence state the exact area of the triangle.

Answer .... cm^2 [1]

(iv) Solve $h^2 + 7h - 240 = 0$, giving your answers correct to 1 decimal place.

Answer h = ... or .... [3]

(v) Calculate the perimeter of the triangle.

Answer .... cm [1]

[June/2015/P22/Q9(b,c)]

---

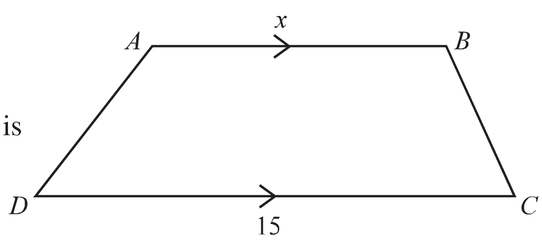
3. Solve $\dfrac{4}{x} + \dfrac{2}{x+2} = 3$.

Answer x = ... or .... [3]

[Nov/2015/P21/Q5(b)]

---

4. (a) *ABCD* is a trapezium with *AB* parallel to *DC*.
DC = 15 cm and *AB* = x cm.
The perpendicular distance between *AB* and *DC* is 3 cm less than the length of *AB*.
The area of *ABCD* is 75 cm^2.



(i) Show that $x^2 + 12x - 195 = 0$.

[2]

(ii) Find *AB*, giving your answer correct to 1 decimal place.

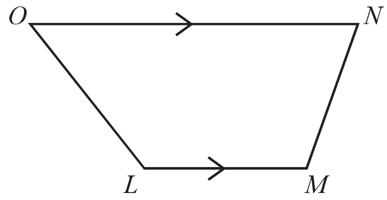
Answer .... cm [3]

(iii) *AD* is 0.8 cm longer than *BC*.

Given that the perimeter of the trapezium is 38.0 cm, calculate *AD*.

Answer .... cm [2]

(b) Another trapezium, *LMNO*, has *LM* parallel to *ON*. The reflex angle *LMN* = 252°.



(i) Calculate $M\hat{N}O$.

Answer .... [2]

(ii) The ratios of the angles inside the trapezium are $L\hat{O}N : L\hat{M}N = 1:2$ and $O\hat{L}M : M\hat{N}O = 1:k$.

Find *k*, giving your answer as a fraction in its simplest form.

Answer .... [3]

[Nov/2015/P21/Q10]

---

5. (a) (i) Solve the equation $\left(x+\frac{7}{2}\right)=\pm\frac{\sqrt{5}}{2}$.

Give both answers correct to 2 decimal places.

Answer *x* = ... or .... [2]

(ii) The solutions of $\left(x+\frac{7}{2}\right)=\pm\frac{\sqrt{5}}{2}$ are also the solutions of $x^2+Bx+C=0$,

where *B* and *C* are integers.

Find *B* and *C*.

Answer *B* = ... *C* = ... [3]

(b) Solve the inequality $7-3x>13$.

Answer *x* .... [2]

(c) Factorise $6x - 3yt + 18y - xt$.

Answer .... [2]

(d) Solve these simultaneous equations.

$$3a + 4b = -13$$
$$5a + 6b = -11$$

Answer a = ...

b = ... [4]

[Nov/2015/P22/Q6]

---

6. (a) Solve the equation $\dfrac{p-1}{7-p} = 5$.

Answer .... [2]

(b) (i) Factorise $4t^2 + 35t - 9$.

Answer .... [2]

(ii) Hence solve the equation $4t^2 + 35t - 9 = 0$.

Answer .... [1]

[June/2016/P21/Q2(a,d)]

---

7. (a) $$p = \frac{8 - 5q}{q}$$

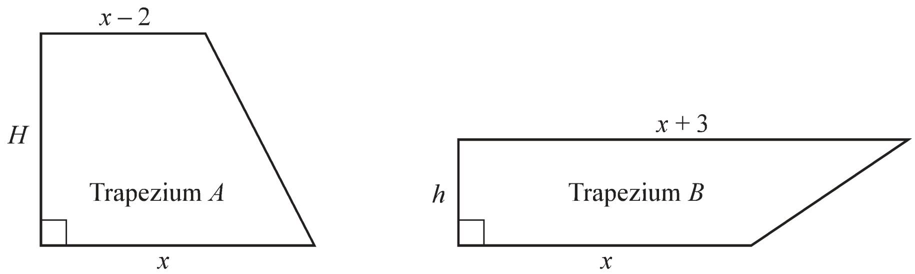
(i) Find p when $q = 2.6$.

Answer .... [1]

(ii) Express q in terms of p.

Answer .... [2]

(b)



The lengths of the parallel sides of trapezium A are x cm and $(x - 2)$ cm.
The lengths of the parallel sides of trapezium B are x cm and $(x + 3)$ cm.
The height of trapezium A is H cm and the height of trapezium B is h cm.
The area of each trapezium is 15 cm^2.

(i) Show that $H = \dfrac{15}{x-1}$ and $h = \dfrac{30}{2x+3}$.

[2]

(ii) Find an expression in terms of x for the difference in height, $H - h$, between trapezium A and trapezium B, and show that it simplifies to $\dfrac{75}{(x-1)(2x+3)}$.

[3]

(iii) The difference in height is 1.5 cm.

(a) Show that $2x^2 + x - 53 = 0$.

[2]

(b) Find x, giving your answer correct to 2 decimal places.

Answer x = ... [2]

[June/2016/P21/Q8]

8. (a) Factorise fully $8x^2y - 12x^5$.

Answer .... [1]

(b) Solve $4x-2(x+5)=3$.

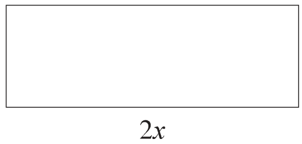
Answer .... [2]

(c) Solve $7-5y<20$.

Answer y .... [2]

(d) A rectangle has length $2x$ cm, perimeter 18 cm and area 10 cm^2.

(i) Show that $2x^2-9x+5=0$.



[2]

(ii) Solve $2x^2-9x+5=0$, giving your answers correct to 2 decimal places.

Answer x = ... or .... [3]

(iii) Find the difference between the length and the width of the rectangle.

Answer .... cm [1]

[June/2016/P22/Q5]

---

9. On Monday, Abdul sold 140 boxes of matches at 30 cents per box.

(a) Calculate the income, in dollars, Abdul received on Monday.

Answer \$ .... [1]

(b) On Tuesday, the price per box decreased by 10% and the number of boxes sold increased by 30%.

Calculate the percentage change in the income.

Answer .... % [3]

(c) On Wednesday, the price of a box was y cents less than it was on Monday.
Abdul sold $4y$ more boxes on Wednesday than he did on Monday.

(i) Write down an expression, in terms of y, for the income received on Wednesday.
Give your answer in dollars.

Answer \$ .... [2]

(ii) Given that this income is equal to \$40, write down an equation in y and show that it simplifies to

$$y^2 + 5y - 50 = 0.$$

[2]

(iii) Solve the equation $y^2 + 5y - 50 = 0$.

Answer $y =$ .... or .... [3]

(iv) Hence find the number of boxes sold on Wednesday.

Answer .... [1]

[Nov/2016/P21/Q9]

---

10. (i) Find the two solutions of $5x - 1 = \pm 9$.

Answer $x =$ .... or .... [2]

(ii) The solutions of $5x - 1 = \pm 9$ are also the solutions of $5x^2 + Bx + C = 0$, where B and C are integers. Find B and C.

Answer $B =$ .... , $C =$ .... [2]

[Nov/2016/P22/Q2(e)]

---

11. (a) $x = \sqrt{a^2 + b^2}$

(i) Calculate x when $a = -0.73$ and $b = 1.84$.

Answer .... [1]

(ii) Express b in terms of x and a.

Answer $b =$ .... [2]

ANSWERS

Topic 9 - Solutions of Equations

1. (a) $(x-1)(x^2+x+1)$
$= x^3+x^2+x-x^2-x-1 = x^3-1$

(b) $\dfrac{3x}{x+2}-\dfrac{4}{x-2}=3$
$\Rightarrow \dfrac{3x(x-2)-4(x+2)}{(x+2)(x-2)}=3$
$\Rightarrow 3x^2-10x-8=3(x^2-4) \quad \Rightarrow \quad x=\dfrac{2}{5}$

(c) $4x-3y=4$ .....(1), $\quad 3x-4y=6.5$ .....(2)
$(1)\times 3: \quad 12x-9y=12$ ....(3)
$(2)\times 4: \quad 12x-16y=26$ ....(4)
$(3)-(4)$ gives, $\quad y=-2$
Subst. $y=-2$ into (1) to get, $\quad x=-0.5$

2. (a) $\dfrac{7}{3-2m}=4 \quad \Rightarrow \quad 7=12-8m \quad \Rightarrow \quad m=\dfrac{5}{8}$

(b) (i) By Pythagoras, $(7+h)^2+h^2=(23)^2$
$\Rightarrow (49+14h+h^2)+h^2=529$
$\Rightarrow 2h^2+14h-480=0$
$\Rightarrow h^2+7h-240=0$

(ii) Area $=\dfrac{1}{2}(7+h)(h)=\dfrac{1}{2}(7h+h^2)$ cm^2

(iii) From (b) (i), $h^2+7h=240$
$\therefore$ Area of $\Delta=\dfrac{1}{2}(240)=120$ cm^2

(iv) $h=\dfrac{-7\pm\sqrt{(7)^2-4(1)(-240)}}{2(1)}$
$=\dfrac{-7\pm\sqrt{1009}}{2}$
$\therefore \quad h=12.4$ or -19.4

(v) Perimeter $=h+h+7+23$
$=12.38+12.38+7+23$
$=54.76\approx 54.8$ cm

3. $\dfrac{4}{x}+\dfrac{2}{x+2}=3$
$\Rightarrow \dfrac{4(x+2)+2x}{x(x+2)}=3 \quad \Rightarrow \quad 6x+8=3(x^2+2x)$
$\Rightarrow 3x^2=8 \quad \Rightarrow \quad x=\pm\sqrt{\dfrac{8}{3}}$

4. (a) (i) Area of trapezium $=75$ cm^2
$\Rightarrow \dfrac{1}{2}(x+15)(x-3)=75$
$\Rightarrow x^2-3x+15x-45=150$
$\Rightarrow x^2+12x-195=0$

(ii) $x=\dfrac{-12\pm\sqrt{(12)^2-4(1)(-195)}}{2(1)}$
$=\dfrac{-12\pm\sqrt{924}}{2}$
$\therefore x=9.2$ or -21.2, so $AB=9.2$ cm

(iii) Let $BC=y$ cm
$\therefore \quad 9.2+y+15+(y+0.8)=38$
$\Rightarrow 2y+25=38 \quad \Rightarrow \quad y=6.5$
$\therefore \quad AD=6.5+0.8=7.3$ cm

(b) (i) Obtuse $L\widehat{M}N=360°-252°=108°$
$\therefore \quad M\widehat{N}O=180°-108°=72°$

(ii) $L\widehat{O}N=\dfrac{1}{2}(108°)=54°$
$O\widehat{L}M=360°-(108°+72°+54°)$
$=126°$
Given, $\dfrac{O\widehat{L}M}{M\widehat{N}O}=\dfrac{1}{k}$
$\Rightarrow \dfrac{126°}{72°}=\dfrac{1}{k} \quad \Rightarrow \quad k=\dfrac{72°}{126°}=\dfrac{4}{7}$

5. (a) (i) $\left(x+\dfrac{7}{2}\right)=\pm\dfrac{\sqrt{5}}{2}$
$\Rightarrow x+\dfrac{7}{2}=\dfrac{\sqrt{5}}{2}$ or $x+\dfrac{7}{2}=-\dfrac{\sqrt{5}}{2}$
$\therefore \quad x=-2.38$ or -4.62

(ii) $\left(x+\frac{7}{2}\right)^2=\left(\pm\frac{\sqrt{5}}{2}\right)^2$

$\Rightarrow\ x^2+7x+\frac{49}{4}=\frac{5}{4}$

$\Rightarrow\ x^2+7x+11=0$

$\therefore\ B=7,\ C=11$

(b) $7-3x>13 \Rightarrow -3x>6 \Rightarrow x<-2$

(c) Re-arrange as, $6x-xt-3yt+18y$

$=x(6-t)-3y(t-6)$

$=x(6-t)+3y(6-t)=(6-t)(x+3y)$

(d) $3a+4b=-13$.....(1), $5a+6b=-11$....(2)

$(1)\times3$: $9a+12b=-39$ ....(3)

$(2)\times2$: $10a+12b=-22$ ....(4)

$(3)-(4)$ gives, $a=17$

Subst. $a=17$ into (1) to get, $b=-16$

6. **(a)** $\frac{p-1}{7-p}=5 \Rightarrow p-1=35-5p \Rightarrow p=6$

(b) **(i)** $4t^2+35t-9=(4t-1)(t+9)$

(ii) Using (b) (i), $(4t-1)(t+9)=0$

$\Rightarrow t=\frac{1}{4}$ or $t=-9$

7. **(a)** **(i)** $p=\frac{8-5(2.6)}{2.6}=-1.92$

(ii) $p=\frac{8-5q}{q}$

$\Rightarrow qp=8-5q \Rightarrow q=\frac{8}{p+5}$

(b) **(i)** Area of trapezium $A=15\text{ cm}^2$

$\Rightarrow \frac{1}{2}H(x+x-2)=15 \Rightarrow H=\frac{15}{x-1}$

For trap. B, $\frac{1}{2}h(x+x+3)=15$

$\Rightarrow h(2x+3)=30 \Rightarrow h=\frac{30}{(2x+3)}$

(ii) $H-h=\frac{15}{x-1}-\frac{30}{(2x+3)}$

$=\frac{15(2x+3)-30(x-1)}{(x-1)(2x+3)}$

$=\frac{75}{(x-1)(2x+3)}$

(iii) **(a)** $\frac{75}{(x-1)(2x+3)}=1.5$

$\Rightarrow 75=1.5(2x^2+x-3)$

$\Rightarrow 2x^2+x-53=0$

(b) $x=\frac{-1\pm\sqrt{(1)^2-4(2)(-53)}}{2(2)}$

$=\frac{-1\pm\sqrt{425}}{4}$, $\therefore\ x=4.90$

8. **(a)** $8x^2y-12x^5=4x^2(2y-3x^3)$

(b) $4x-2(x+5)=3$

$\Rightarrow 4x-2x-10=3 \Rightarrow x=6.5$

(c) $7-5y<20 \Rightarrow -5y<13 \Rightarrow y>-2.6$

(d) **(i)** Let width of the rectangle be w,

Given perimeter $=18$ cm

$\Rightarrow 2(2x+w)=18 \Rightarrow w=9-2x$

Area, $2x\times w=10$

$\Rightarrow 2x\times(9-2x)=10$

$\Rightarrow 2x^2-9x+5=0$

(ii) $x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(5)}}{2(2)}$

$=\frac{9\pm\sqrt{41}}{4}$, $\therefore\ x=3.85$ or 0.65

(iii) Taking $x=3.85$,

Length $=2(3.85)=7.7$ cm

width $=9-2(3.85)=1.3$ cm

Difference $=7.7-1.3=6.4$ cm.

9. **(a)** $\$0.30\times140=\$42$

(b) New price $=90\%\times0.30=\$0.27$

No. of boxes sold $=130\%\times140=182$

% change $=\frac{(0.27\times182)-42}{42}\times100=17\%$

(c) **(i)** Income $=\$\ \frac{(30-y)(140+4y)}{100}$

(ii) $\frac{(30-y)(140+4y)}{100}=40$

$\Rightarrow 4200-20y-4y^2=4000$

$\Rightarrow y^2+5y-50=0$

(iii) $y^2+5y-50=0$

$\Rightarrow (y-5)(y+10)=0$

$\Rightarrow y=5$ or $y=-10$

(iv) No. of boxes sold $=140+4(5)=160$

10. (i) Either, $5x-1=9$ or $5x-1=-9$

$\Rightarrow x=2$ or $-\dfrac{8}{5}$

(ii) $(5x-1)^2=81$

$\Rightarrow 25x^2-10x-80=0$ (divide by 5)

$\Rightarrow 5x^2-2x-16=0$

$\therefore\ B=-2,\quad C=-16$

11. (a) (i) $x=\sqrt{(-0.73)^2+(1.84)^2}=1.98$

(ii) $x=\sqrt{a^2+b^2} \quad\Rightarrow\quad b^2=x^2-a^2$

$\Rightarrow b=\sqrt{(x+a)(x-a)}$

(b) (i) $PQ=\dfrac{17}{x+5}$ cm

(ii) $AB=(\dfrac{17}{x+5}+3)$ cm

Area of $ABCD=17$

$\Rightarrow x\times(\dfrac{17}{x+5}+3)=17$

$\Rightarrow \dfrac{17x+3x(x+5)}{x+5}=17$

Simplify to get, $3x^2+15x-85=0$

(iii) $x=\dfrac{-15\pm\sqrt{(15)^2-4(3)(-85)}}{2(3)}$

$=\dfrac{-15\pm\sqrt{1245}}{6}$

$\therefore\ x=3.38$ or -8.38

(iv) $PQ=\dfrac{17}{3.38+5}=2.03$ cm

$PS=3.38+5=8.38$ cm

Perimeter $=2(2.03+8.38)=20.82$

12. (i) $4x^2+12x+9=(2x+3)^2$

(ii) $4x^2+12x+9=49$

$\Rightarrow (2x+3)^2=49 \quad\Rightarrow\quad (2x+3)=\pm7$

$\therefore\ x=2$ or -5

13. $3x^2-x-5=0$. By quadratic formula,

$x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(3)(-5)}}{2(3)}=\dfrac{1\pm\sqrt{61}}{6}$

$\therefore\ x=1.47$ or -1.14

14. (a) $\dfrac{y}{2y+3}=\dfrac{2}{y+5}$

$\Rightarrow y(y+5)=2(2y+3)$

$\Rightarrow y^2+y-6=0 \Rightarrow (y+3)(y-2)=0$

$\therefore\ y=-3$ or 2

(b) $p=\dfrac{4t+1}{2-t}$

$\Rightarrow 2p-pt=4t+1$

$\Rightarrow 4t+pt=2p-1 \quad\Rightarrow\quad t=\dfrac{2p-1}{4+p}$

15. (a) 3000 litres —— 12 minutes

1750 litres —— $\dfrac{12}{3000}\times1750=7$ minutes

(b) (i) $\dfrac{2500}{x}$ minutes

(ii) Time for large pump $=\dfrac{2500}{x+20}$ minutes

$\dfrac{2500}{x}-\dfrac{2500}{x+20}=15$

$\Rightarrow 2500\left(\dfrac{x+20-x}{x(x+20)}\right)=15$

$\Rightarrow 50000=15x(x+20)$

Simplify to, $3x^2+60x-10000=0$

(iii) $x=\dfrac{-60\pm\sqrt{(60)^2-4(3)(-10000)}}{2(3)}$

$=\dfrac{-60\pm\sqrt{123600}}{6}$

$\therefore\ x=48.59$ or -68.59

(iv) Time for larger pump $=\dfrac{2500}{48.59+20}$

$=36.448$ minutes

$=36$ minutes 27 seconds

16. (a) $2x(x+1)=3(4-x)$

$\Rightarrow 2x^2+5x-12=0$

$\Rightarrow (2x-3)(x+4)=0 \quad \therefore\ x=\dfrac{3}{2}$ or -4

(b) (i) Let p be cost of a pen and n be the cost of a notebook.

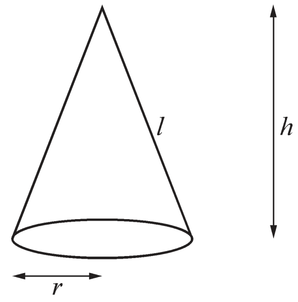
$\therefore\ 3p+2n=4.8\ \ \&\ \ 5p+4n=9$

TOPIC 14

Graphs of Functions

1. **[Curved surface area of a cone = πrl]**

The diagram shows a solid cone with radius r cm, height h cm and slant height l cm.
Suleman makes some solid cones. The slant height of each of his cones is 4 cm more than its radius.
Use $\pi = 3$ throughout this question.



(a) Show that the **total** surface area, A cm^2, of each of Suleman's cones is given by $A = 6r(r+2)$.

[2]

(b) Complete the table for $A = 6r(r+2)$.

r	0	1	2	3	4	5	6
A	0	18			144	210	288

[1]

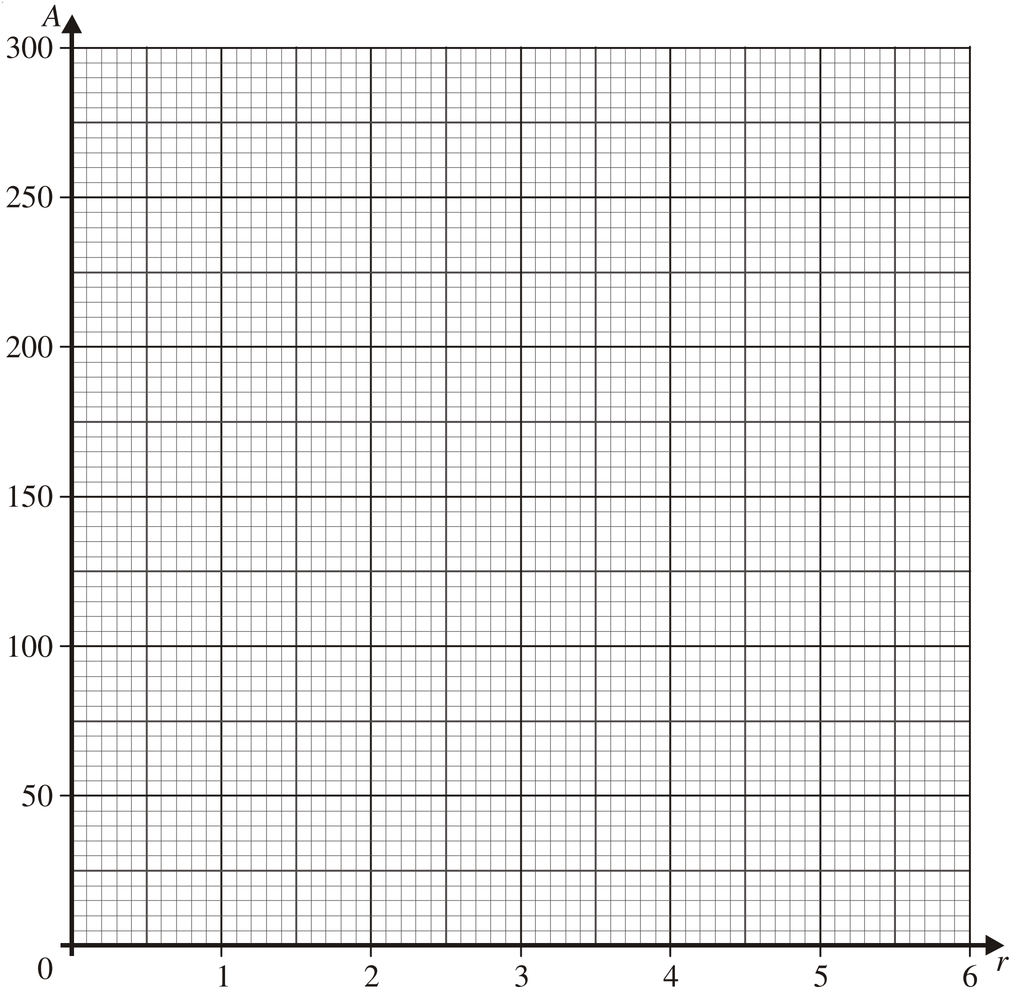
(c) On the grid below, draw the graph of $A = 6r(r+2)$. [2]

(d) Find an expression for h in terms of r.

Answer h = ... [2]

(e) The height of one of Suleman's cones is 12 cm.
Calculate its radius.

Answer .... cm [2]

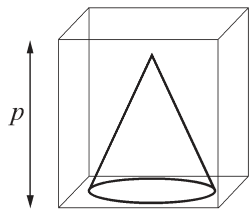


(f) Another of Suleman's cones has a surface area of 200 cm^2.

(i) Use your graph to find the radius of this cone.

Answer .... cm [1]

(ii) This cone is placed in a box of height p cm, where p is an integer.
Find the smallest possible value of p.



Answer p = ... [2]

[Nov/2015/P21/Q8]

2. The distance, d metres, of a moving object from an observer after t minutes is given by

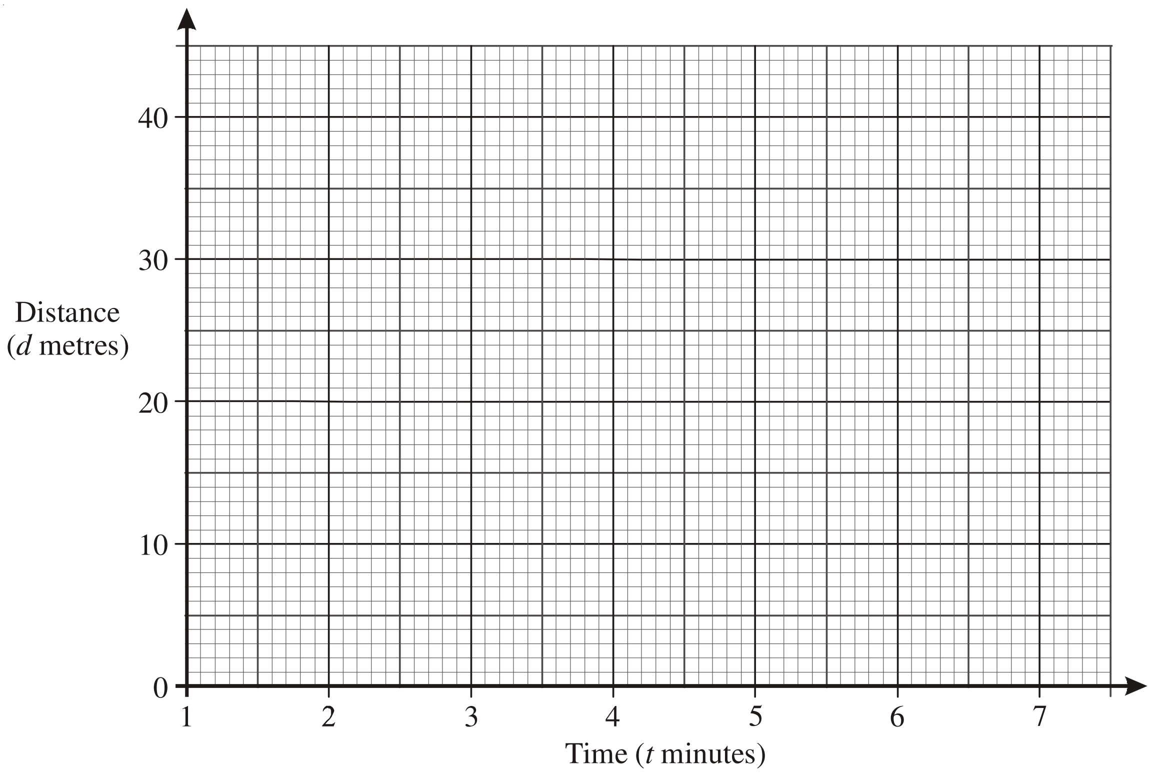
$$d = t^2 + \frac{48}{t} - 20.$$

(a) Some values of t and d are given in the table.
The values of d are given to the nearest whole number where appropriate.

t	1	1.5	2	2.5	3	3.5	4	4.5	5	6	7
d	29	14	8	5	5	6	8	11	15	24	

Complete the table. [1]

(b) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

(c) (i) By drawing a tangent, calculate the gradient of the curve when $t = 4$.

Answer .... [2]

(ii) Explain what this gradient represents.

Answer .... [1]

(d) For how long is the object less than 10 metres from the observer?

Answer .... minutes [2]

(e) (i) Using your graph, write down the two values of t when the object is 12 metres from the observer.
For each value of t, state whether the object is moving towards or away from the observer.

Answer When t = ..., the object is moving .... the observer.

When t = ..., the object is moving .... the observer. [2]

(ii) Write down the equation that gives the values of t when the object is 12 metres from the observer.

Answer .... [1]

(iii) This equation is equivalent to $t^3 + At + 48 = 0$.

Find A.

Answer A = ... [1]

[Nov/2015/P22/Q9]

---

3. The table below is for $y = x^2 + x - 3$.

x	–3	–2	–1	0	1	2
y	3	–1	–3	–3	–1	3

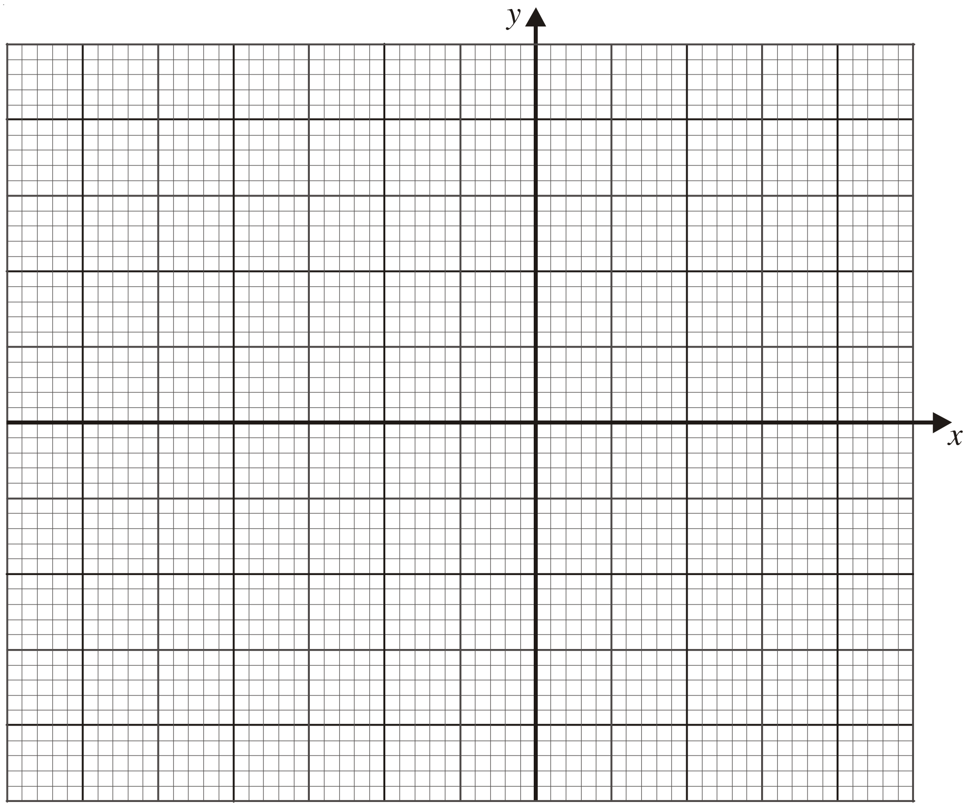
(a) Using a scale of 2 cm to 1 unit on the x-axis for $-3 \le x \le 2$
and a scale of 1 cm to 1 unit on the y-axis for $-4 \le y \le 4$,
plot the points from the table and join them with a smooth curve. [2]

(b) (i) Use your graph to estimate the solutions of the equation $x^2 + x - 3 = 0$.

Answer x = ... or .... [1]

(ii) Use your graph to estimate the solutions of the equation $x^2 + x - 5 = 0$.

Answer x = ... or .... [2]



(c) By drawing a tangent, estimate the gradient of the curve at (1, –1).

Answer .... [2]

(d) The equation $x^2 - x - 1 = 0$ can be solved by drawing a straight line on the graph of $y = x^2 + x - 3$.

(i) Find the equation of this straight line.

Answer .... [2]

(ii) Draw this straight line and hence solve $x^2 - x - 1 = 0$.

Answer x = ... or .... [2]

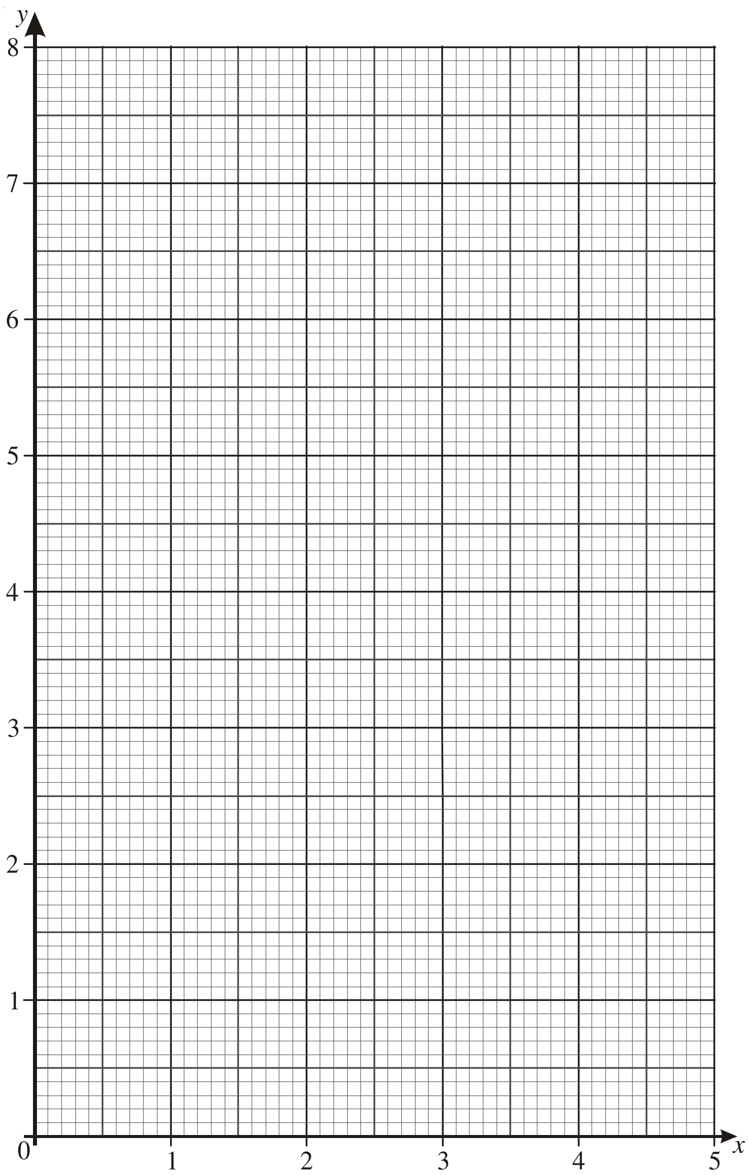
[June/2016/P21/Q3]

4. The table below shows some values of x and the corresponding values of y for $y = \frac{1}{4} \times 2^x$.

x	0	1	2	3	4	5
y	$\frac{1}{4}$		1	2	4	8

(a) Complete the table. [1]

(b) On the grid below, draw the graph of $y = \frac{1}{4} \times 2^x$. [2]



(c) By drawing a suitable line, find the gradient of your graph where $x = 4$.

Answer .... [2]

(d) (i) Show that the line $2x + y = 6$, together with the graph of $y = \frac{1}{4} \times 2^x$, can be used to solve the equation $2^x + 8x - 24 = 0$.

[1]

(ii) Hence solve $2^x + 8x - 24 = 0$.

Answer x = ... [2]

(e) The points P and Q are (2, 3) and (5, 4) respectively.

(i) Find the gradient of PQ.

Answer .... [1]

(ii) On the grid, draw the line l, parallel to PQ, that touches the curve $y = \frac{1}{4} \times 2^x$. [1]

(iii) Write down the equation of l.

Answer .... [2]

[June/2016/P22/Q8]

5.

$$y = \frac{3}{5} \times 2^x$$

The table shows some values of x and the corresponding values of y, correct to one decimal place where necessary.

x	−1.5	−1	0	1	2	2.5	3	3.5	4
y	p	0.3	0.6	1.2	2.4	3.4	4.8	6.8	9.6

(a) Calculate p.

Answer .... [1]

(b) On the grid,

- using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 4$,
- using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \leq y \leq 10$,
- plot the points from the table and join them with a smooth curve. [3]

ANSWERS

Topic 14 - Graphs of Functions

1. **(a)** $A = \pi r^2 + \pi r l$

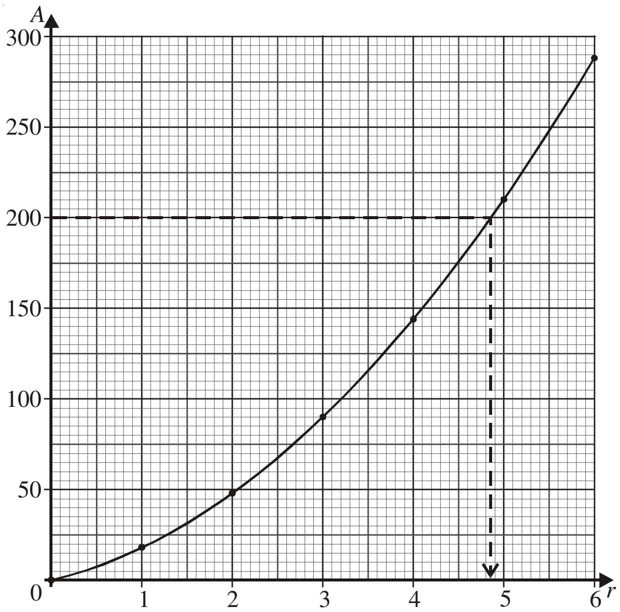
$\Rightarrow \; A = 3r^2 + 3r(r+4)$

$\Rightarrow \; A = 6r^2 + 12r \;\; \Rightarrow \;\; A = 6r(r+2).$

(b) When $r = 2, \;\; A = 6(2)(2+2) = 48$

When $r = 3, \;\; A = 6(3)(3+2) = 90$

(c)



(d) Using Pythagoras, $h = \sqrt{(r+4)^2 - r^2}$

$\Rightarrow \; h = \sqrt{r^2 + 8r + 16 - r^2}$

$\Rightarrow \; h = \sqrt{8r + 16}$

(e) $\sqrt{8r+16} = 12$

$\Rightarrow \; 8r + 16 = 144 \;\; \Rightarrow \;\; r = 16$ cm

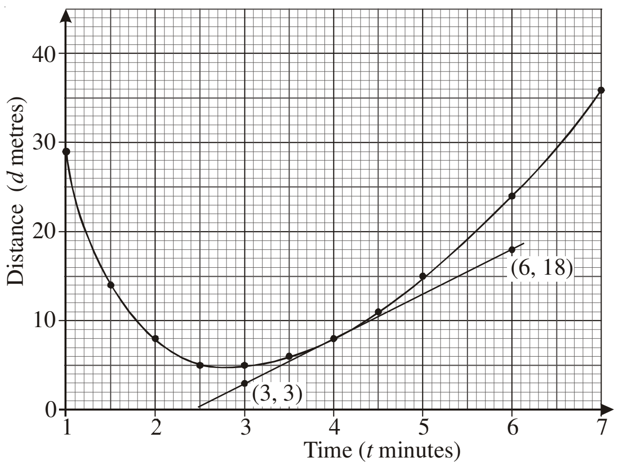
(f) **(i)** $r = 4.85$ cm

(ii) $h = \sqrt{8(4.85) + 16} = 7.40$

$\therefore$ smallest value of $p = 8$ cm

2. **(a)** When $t = 7, \;\; d = (7)^2 + \frac{48}{7} - 20 \approx 36$

(b)



(c) **(i)** Using (3, 3) and (6, 18) on tangent,

gradient $= \frac{18-3}{6-3} = 5$

(ii) The gradient represents the speed of the object at 4 minutes.

(d) At $d = 10$ m, $\;\; t = 1.8$ & 4.4 min.

Length of time $= 4.4 - 1.8 = 2.6$ min.

(e) **(i)** When t = ...**1.63**..., the object is moving ...**towards**.... the observer.

When t = ...**4.65**..., the object is moving ...**away from**. the observer.

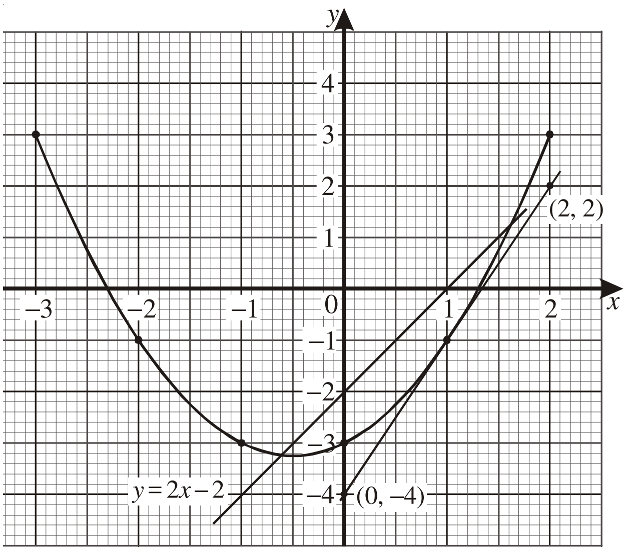
(ii) $t^2 + \frac{48}{t} - 20 = 12$

$\Rightarrow \; t^2 + \frac{48}{t} - 32 = 0$

(iii) $t^2 + \frac{48}{t} - 32 = 0$

$\Rightarrow \; t^3 - 32t + 48 = 0, \quad \therefore \;\; A = -32$

3. **(a)**



(b) **(i)** $x^2 + x - 3 = 0 \quad \Rightarrow \quad y = 0$

$\therefore$ from graph, $x = -2.3$ or 1.3

(ii) $x^2 + x - 5 = 0$

$\Rightarrow \; x^2 + x - 3 - 2 = 0 \quad \Rightarrow \quad y = 2$

$\therefore$ from graph, $x = -2.8$ or 1.8

(c) Using $(0, \; -4)$ & $(2, 2)$ on the tangent,

gradient $= \dfrac{2-(-4)}{2-0} = 3$

(d) **(i)** $x^2 - x - 1 = 0$

$\Rightarrow \; x^2 - x - 1 + 2x - 2 = 2x - 2$

$\Rightarrow \; x^2 + x - 3 = 2x - 2$

$\Rightarrow \; y = 2x - 2$

(ii) Refer to graph for $y = 2x - 2$.

The graphs meet at, $x = -0.6$, or 1.6

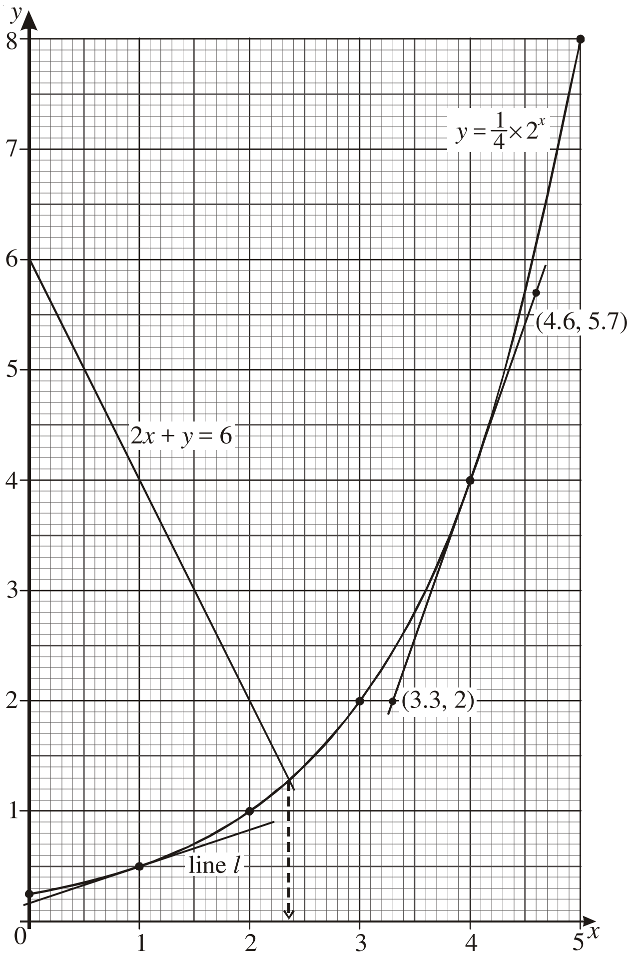
(ii) From graph, $x = 2.35$

(e) **(i)** Gradient of $PQ = \dfrac{4-3}{5-2} = \dfrac{1}{3}$

(ii) Refer to graph.

(iii) From graph, y-intercept of $l = 0.17$

$\therefore$ equation of l: $y = \dfrac{1}{3}x + 0.17$



4. **(a)** When $x = 1$, $y = \dfrac{1}{4} \times 2^1 = \dfrac{1}{2}$

(b) Refer to graph.

(c) Taking $(3.3, \; 2)$ and $(4.6, \; 5.7)$ on the tangent, gradient $= \dfrac{5.7-2}{4.6-3.3} = 2.85$

(d) **(i)** Substitute eq. of line into curve,

$2x + (\dfrac{1}{4} \times 2^x) = 6$

$\Rightarrow \; \dfrac{1}{4} \times 2^x = 6 - 2x$

$\Rightarrow \; 2^x + 8x - 24 = 0$

5. **(a)** $p = \dfrac{3}{5} \times 2^{-1.5} = 0.21$

(b) Refer to graph on next page.

(c) Taking $(1.3, 0.6)$ and $(3.6, 6)$ on the tangent, gradient $= \dfrac{6-0.6}{3.6-1.3} = 2.35$

(d) **(i)** Refer to graph.

(ii) Gradient $= \dfrac{3.6-0}{2+0.4} = 1.5$

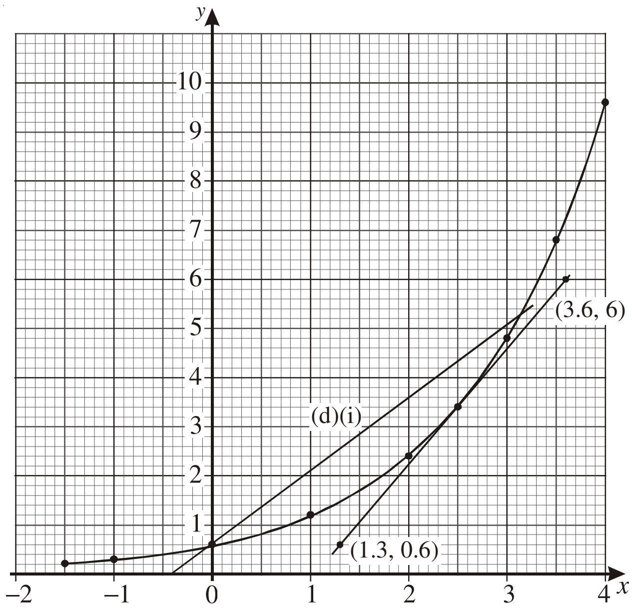
Equation, $y = 1.5x + 0.6$

(iii) $x = 0$ and $x = 3.15$

(iv) Subst. line into curve,

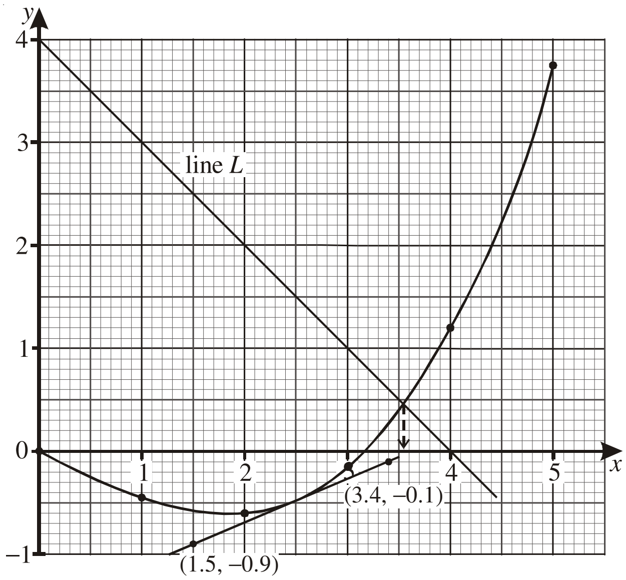
$$\frac{3}{5}\times 2^x = 1.5x + 0.6$$

$$\Rightarrow\ 2^x = 2.5x + 1. \quad \therefore\ A = 2.5,\ \ B = 1$$



6. **(a)** When $x = 5$, $\quad y = \frac{5}{20}(5^2 - 10) = 3.75$

(b)



(c) Using (1.5, −0.9) and (3.4, −0.1),

$$\text{gradient} = \frac{-0.1-(-0.9)}{3.4-1.5} = 0.421$$

(d) $\frac{x}{20}(x^2 - 10) = 0 \quad \Rightarrow \quad y = 0$

From graph, at $y = 0$, $\quad x = 3.17$

(e) **(i)** $x^3 + 10x - 80 = 0$

$$\Rightarrow\ x^3 = 80 - 10x$$

$$\Rightarrow\ x^3 - 10x = 80 - 20x$$

$$\Rightarrow\ x(x^2 - 10) = 20(4 - x)$$

$$\Rightarrow\ \frac{x}{20}(x^2 - 10) = 4 - x$$

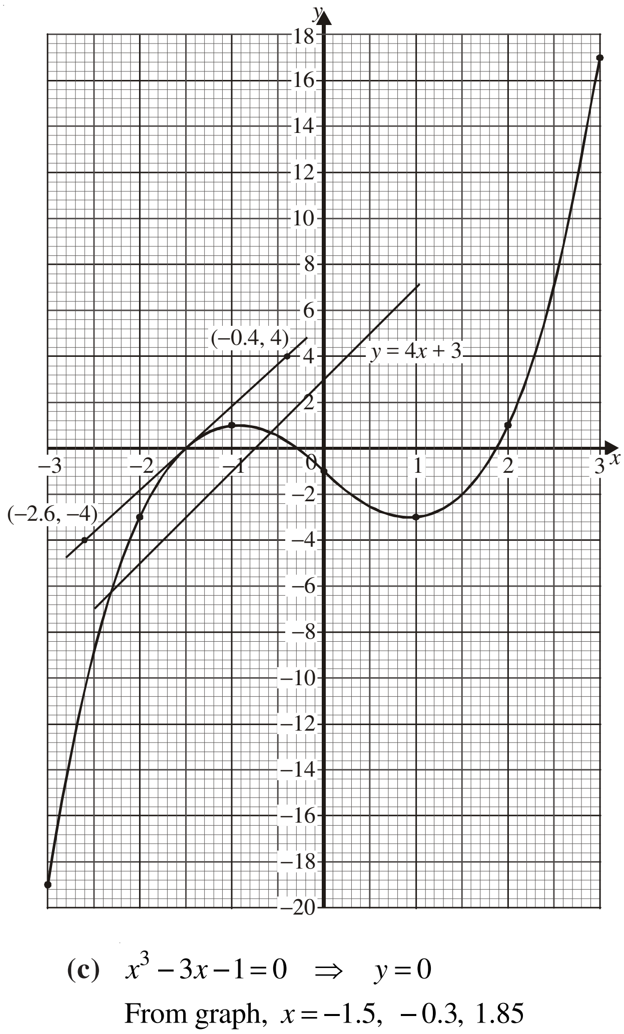
$$\Rightarrow\ y = 4 - x$$

(ii) Refer to graph on next page.

(iii) From graph, $\quad x = 3.55$

7. **(a)** For $x = 3$, $\quad y = 3^3 - 3(3) - 1 = 17$

(b)



(c) $x^3 - 3x - 1 = 0 \quad \Rightarrow \quad y = 0$

From graph, $x = -1.5,\ -0.3,\ 1.85$

(d) Using (−2.6, −4) and (−0.4, 4),

$$\text{gradient} = \frac{4-(-4)}{-0.4-(-2.6)} = 3.64$$

(e) **(i)** Refer to graph.

TOPIC 23

Bearings and Trigonometry

1. (a) (i) Evaluate $\dfrac{8\sin 54°}{\sin 18°}$.

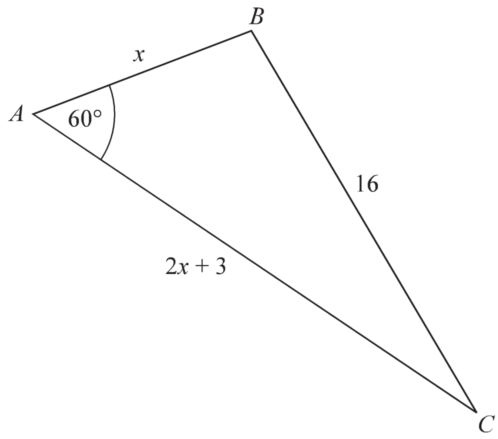
Answer .... [1]

(ii) Evaluate $\sqrt{4.73^2 - 1.65\sin 43°}$.

Answer .... [1]

(b) In the triangle ABC, BC = 16 cm and $B\hat{A}C = 60°$.
$AB = x$ cm and $AC = 2x + 3$ cm.

(i) Form an equation in x and show that it simplifies to $3x^2 + 9x - 247 = 0$.



[4]

(ii) Solve the equation $3x^2 + 9x - 247 = 0$, giving your answers correct to 2 decimal places.

Answer x = ... or .... [3]

(iii) Hence write down the lengths of AB and AC.

Answer AB = ... cm AC = ... cm [1]

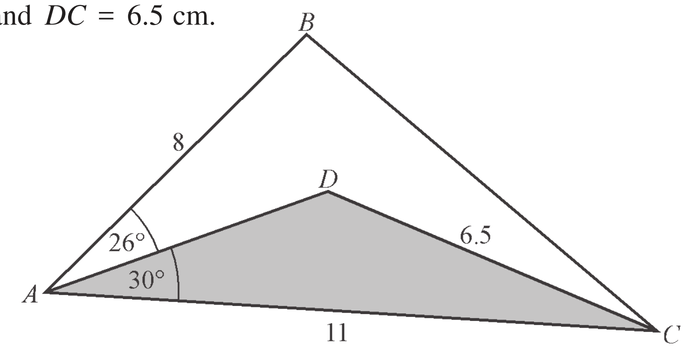
(iv) Find the area of triangle ABC.

Answer .... cm^2 [2]

[June/2015/P21/Q7]

---

2. In the diagram, $AB = 8$ cm, $AC = 11$ cm and $DC = 6.5$ cm. $B\hat{A}D = 26°$ and $D\hat{A}C = 30°$.



(a) Calculate BC.

Answer .... cm [4]

(b) Calculate the obtuse angle ADC.

Answer .... [3]

(c) Find the percentage of triangle ABC that has been shaded.

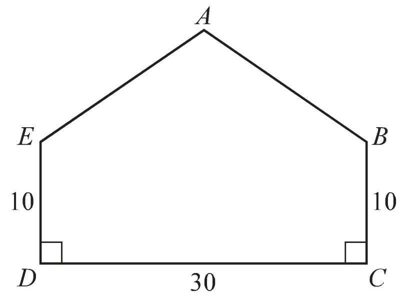
Answer .... % [4]

[June/2015/P22/Q5]

3. (a) *ABCDE* is a pentagon with one line of symmetry.

$BC = DE = 10$ cm, $DC = 30$ cm and $B\hat{C}D = C\hat{D}E = 90°$.

The shortest distance between *A* and *DC* is 22 cm.



(i) Calculate *AB*.

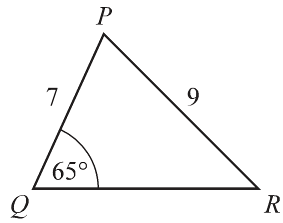
Answer .... cm [2]

(ii) Calculate $A\hat{B}C$.

Answer .... [3]

(b) In triangle *PQR*, $PQ = 7$ cm, $PR = 9$ cm and $P\hat{Q}R = 65°$.

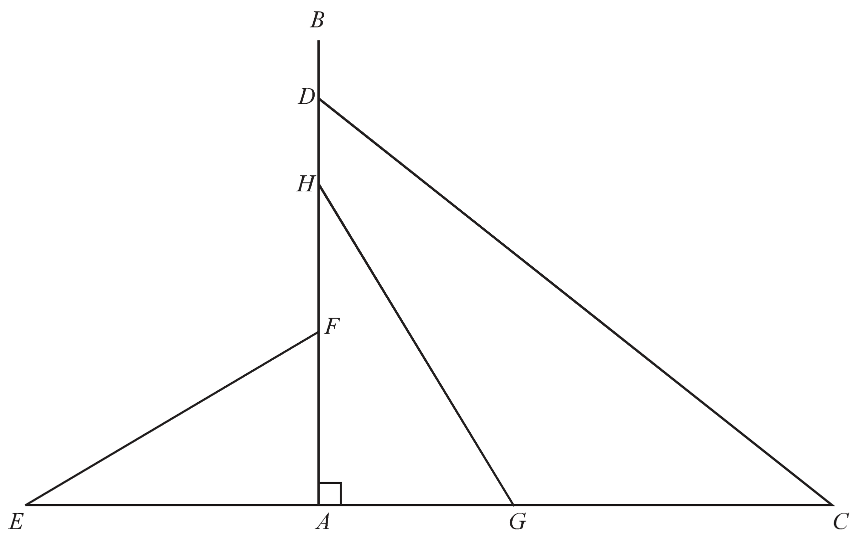
Calculate $P\hat{R}Q$.



Answer .... [3]

[Nov/2015/P21/Q2]

4.



The diagram shows a vertical radio mast, *AB*.
Three of the wires that hold the mast in place are attached to it at *F*, *H* and *D*.
The base *A* of the mast, and the ends *E*, *G* and *C* of the wires are in a straight line on horizontal ground.

(a) The wire *CD* has length 65 m. It is attached to the mast at *D* where $AD = 40$ m.
Calculate *AC*.

Answer .... m [2]

(b) The wire *EF* makes an angle of 25° with the horizontal and is of length 30 m.
Calculate *AF* .

Answer .... m [2]

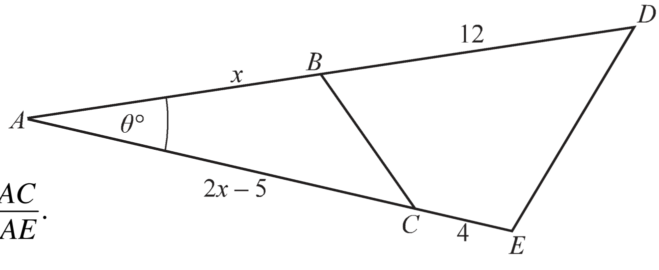
(c) $AH = 35$ m.
The wire *HG* makes an angle of 30° with the mast *AB*.
Calculate *HG*.

Answer .... m [3]

[Nov/2015/P22/Q5]

---

5. *ABD* and *ACE* are straight lines.
BD = 12 cm and *CE* = 4 cm.
AB = *x* cm and *AC* = (2*x* – 5) cm.
Angle $BAC = \theta°$.



(a) Show that $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{AB \times AC}{AD \times AE}$.

[2]

(b) It is given that $\frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{1}{3}$.

Using the result from part **(a)**, form an equation in x and show that it simplifies to $2x^2 - 19x + 6 = 0$.

[3]

(c) (i) Solve the equation $2x^2 - 19x + 6 = 0$, giving your answers correct to 2 decimal places.

Answer x = ... or .... [3]

(ii) State, with a reason, which of these solutions does **not** apply to triangle ABC.

Answer .... [1]

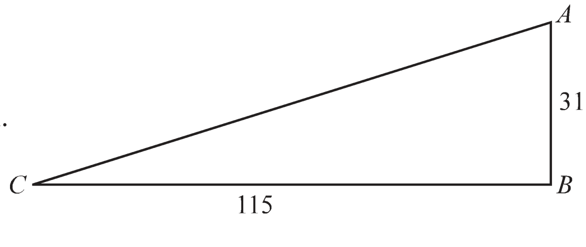
(d) Given that $\theta = 25$, calculate BC.

Answer .... cm [3]

[Nov/2015/P22/Q7]

---

6. (a) AB is vertical and CB is horizontal.
AB = 31 m and CB = 115 m.
Calculate the angle of depression of C from A.



Answer .... [3]

ANSWERS

Topic 23 - Bearings and Trigonometry

1. **(a)** **(i)** $\dfrac{8\sin 54°}{\sin 18°} = 20.94$

(ii) $\sqrt{4.73^2 - 1.65\sin 43°} = 4.61$

(b) **(i)** Using cosine rule,

$16^2 = x^2 + (2x+3)^2 - 2(x)(2x+3)\cos 60°$

$256 = x^2 + (4x^2 + 12x + 9) - 2x^2 - 3x$

$\Rightarrow\ 3x^2 + 9x - 247 = 0$

(ii) $x = \dfrac{-9 \pm \sqrt{9^2 - 4(3)(-247)}}{2(3)}$

$\Rightarrow\ x = \dfrac{-9 \pm \sqrt{3045}}{6}$

$\therefore\ x = 7.70$ or -10.70

(iii) $AB = 7.70$ cm

$AC = 2(7.70) + 3 = 18.4$ cm

(iv) Area of $\Delta ABC = \dfrac{1}{2}(7.70)(18.4)\sin 60°$

$= 61.35\ \text{cm}^2$

2. **(a)** In $\Delta\, ABD$, using cosine rule,

$BC = \sqrt{(11)^2 + (8)^2 - 2(11)(8)\cos 56°}$

$= 9.3049 \approx 9.30$ cm

(b) $\dfrac{\sin A\hat{D}C}{11} = \dfrac{\sin 30°}{6.5}\ \Rightarrow\ A\hat{D}C = 57.8°$

Obtuse angle $ADC = 180° - 57.8° = 122°$

(c) $A\hat{C}D = 180° - 122° - 30° = 28°$

Area of $\Delta ADC = \dfrac{1}{2}(11)(6.5)\sin 28°$

$= 16.784\ \text{cm}^2$

Area of $\Delta ABC = \dfrac{1}{2}(11)(8)\sin 56°$

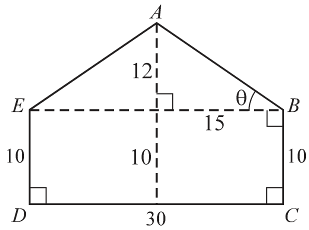
$= 36.478\ \text{cm}^2$

percentage shaded $= \dfrac{16.784}{36.478} \times 100 = 46\%$

3. **(a)** **(i)** AB

$= \sqrt{12^2 + 15^2}$

$= 19.21$ cm



(ii) $\tan\theta° = \dfrac{12}{15}$

$\Rightarrow\ \theta° = 38.7°$

$\therefore\ A\hat{B}C = 38.7° + 90° = 128.7°$

(b) $\dfrac{\sin P\hat{R}Q}{7} = \dfrac{\sin 65°}{9}\ \Rightarrow\ P\hat{R}Q = 44.8°$

4. **(a)** $AC = \sqrt{(65)^2 - (40)^2} = 51.235 \approx 51.2$ m.

(b) $\sin 25° = \dfrac{AF}{30}\ \Rightarrow\ AF = 12.678 \approx 12.7$ m

(c) $\cos 30° = \dfrac{35}{HG}\ \Rightarrow\ HG = 40.4$ m

5. **(a)** $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{\frac{1}{2} \times AB \times AC \times \sin\theta}{\frac{1}{2} \times AD \times AE \times \sin\theta}$

$\Rightarrow\ \dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{AB \times AC}{AD \times AE}$

(b) $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{AB \times AC}{AD \times AE} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{x \times (2x-5)}{(x+12) \times (2x-5+4)} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{2x^2 - 5x}{2x^2 + 23x - 12} = \dfrac{1}{3}$

$\Rightarrow\ 3(2x^2 - 5x) = 2x^2 + 23x - 12$

$\Rightarrow\ 4x^2 - 38x + 12 = 0$

$\Rightarrow\ 2x^2 - 19x + 6 = 0$

(c) **(i)** $x = \dfrac{-(-19) \pm \sqrt{(-19)^2 - 4(2)(6)}}{2(2)}$

$= \dfrac{19 \pm \sqrt{313}}{4}$

$\Rightarrow\ x = 9.17$ or 0.33 (2dp).

(ii) For $x = 0.33$,

$AC = 2(0.33) - 5 = -4.34$

Length cannot be negative, therefore $x = 0.33$ does not apply to ΔABC.

(d) $AB = 9.17, \quad AC = 13.34$

Using cosine rule, BC

$= \sqrt{(9.17)^2 + (13.34)^2 - 2(9.17)(13.34)\cos 25^\circ}$

$= \sqrt{40.312} = 6.35$ cm

6. **(a)** $\tan A\hat{C}B = \dfrac{31}{115} \Rightarrow A\hat{C}B = 15.1^\circ$

$\therefore$ Angle of depression of C from $A = 15.1^\circ$

(b) **(i)** $\sin L\hat{J}K = \dfrac{354}{1100} \Rightarrow L\hat{J}K = 18.8^\circ$

(ii) Bearing of K from $J = 270^\circ - 18.8^\circ$

$= 251.2^\circ$

7. **(a)** **(i)** $AC = \sqrt{(2)^2 + (5)^2} = 5.385 \approx 5.39$ m.

(ii) $\sin 15^\circ = \dfrac{CE}{2} \Rightarrow CE = 0.518$ m.

(iii) By pythagoras, $AE = \sqrt{AC^2 - CE^2}$

$AE = \sqrt{(5.385)^2 - (0.518)^2} = 5.36$ m.

In ΔFAE, $\sin F\hat{A}E = \dfrac{5}{AE}$

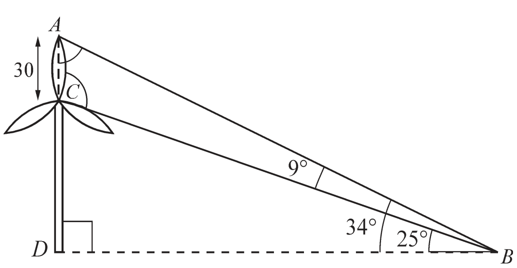
$\Rightarrow \sin F\hat{A}E = \dfrac{5}{5.36} \Rightarrow F\hat{A}E = 68.9^\circ$

(b) **(i)** Using cosine rule,

$\cos\theta = \dfrac{9^2 + 6^2 - 10^2}{2(9)(6)} \Rightarrow \theta = 80.9^\circ$

(ii) $c^2 > (a^2 + b^2)$.

8. **(a)**



$A\hat{C}B$ is an exterior angle to ΔBCD

$\therefore A\hat{C}B = 25^\circ + 90^\circ = 115^\circ$

Using sine rule on ΔABC,

$\dfrac{AB}{\sin 115^\circ} = \dfrac{30}{\sin 9^\circ} \Rightarrow AB = 173.8 \approx 174$ m.

(b) Using cosine rule,

$\cos D\hat{F}E = \dfrac{(75)^2 + (180)^2 - (130)^2}{2(75)(180)}$

$\Rightarrow D\hat{F}E = 38.52^\circ$

Angle of depression of E from F

$= 90^\circ - 38.52^\circ = 51.5^\circ$

(c) **(i)** After one revolution, distance covered by P is the circumference of a circle of radius 30 m. So distance travelled $= 2\pi(30) = 188.495 \approx 188$ m.

(ii) Distance travelled in 15 revolutions

$= 188.495 \times 15 = 2827.425$ m.

Speed of $P = \dfrac{2827.425 \times 60}{1000} \approx 170$ km/h.

(iii) Distance travelled by $Q = 2\pi r$

$\Rightarrow 90 = 2\pi r \Rightarrow r = 14.32$

Length of $PQ = 30 - 14.32 \approx 15.7$ m.

9. **(a)** Bearing of D from $A = 180^\circ + 55^\circ + 48^\circ$

$= 283^\circ$

(b) Bearing of A from $C = 055^\circ$

(c) $A\hat{B}C = 180^\circ - 55^\circ - 51^\circ = 74^\circ$

Using sine rule on ΔABC,

$\dfrac{AB}{\sin 51^\circ} = \dfrac{19}{\sin 74^\circ} \Rightarrow AB = 15.36$ km.

(d) By cosine rule,

$DC = \sqrt{19^2 + 27^2 - 2(19)(27)\cos 48^\circ}$

$= \sqrt{403.472} = 20.1$ km

(e) Draw a line CX from C to DA, such that $CX \perp DA$. Now in ΔACX,

$\cos 48^\circ = \dfrac{AX}{19} \Rightarrow AX = 12.713$ km.

$DX = 27 - 12.713 = 14.287$ km

Time taken from D to $A = 216$ minutes

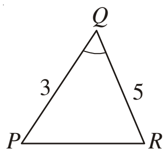
time taken from D to $X = \dfrac{216}{27} \times 14.287$

$= 114$ minutes.

10. (a) (i) $AD = 3\cos 27° = 2.67$ m.

(ii) $\sin 41° = \dfrac{3}{CD} \Rightarrow CD = 4.57$ m

(b) $6 = \dfrac{1}{2}(3)(5)\sin P\hat{Q}R$



$\Rightarrow \sin P\hat{Q}R = \dfrac{12}{15}$

$\Rightarrow P\hat{Q}R = 53.1°$ or $126.9°$

11. (a) In ΔABC, using cosine rule,

$\cos A\hat{C}B = \dfrac{(70)^2 + (110)^2 - (65)^2}{2(70)(110)}$

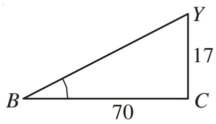
$\Rightarrow A\hat{C}B = 34°$

Bearing of A from $C = 360° - 34° = 326°$

(b) $A\hat{C}D = 180° - 70° - 58° = 52°$

Now, $\dfrac{AD}{\sin 52°} = \dfrac{110}{\sin 70°} \Rightarrow AD = 92.2$ m

(c) (i) Consider ΔBCY,



$\tan C\hat{B}Y = \dfrac{17}{70}$

$\Rightarrow C\hat{B}Y = 13.65°$

$\therefore$ angle of elevation from $B = 13.7°$

(ii) Av. speed $= \dfrac{110}{24}$ m/s

$= \dfrac{110}{24} \times \dfrac{3600}{1000} = 16.5$ km/h

12. (a) (i) $BC = \sqrt{(12)^2 + (8)^2} = 14.42$ km.

(ii) In ΔABD,

$\tan A\hat{D}B = \dfrac{12}{15} \Rightarrow A\hat{D}B = 38.7°$

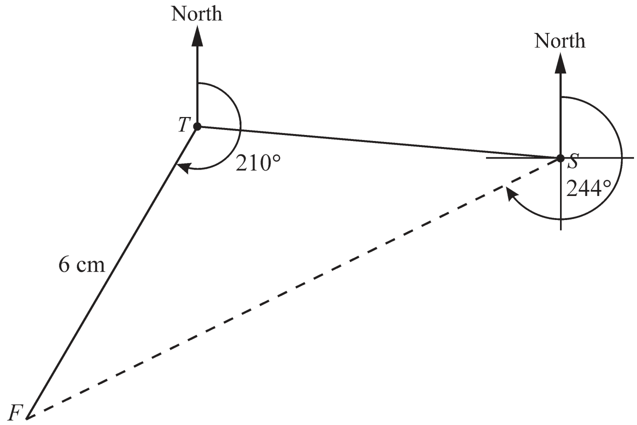
Bearing of A from $D = 90° + 38.7°$

$= 128.7° \approx 129°$

(b) (i) On diagram $TS = 6.4$ cm

Actual distance $= 6.4 \times 75 = 480$ m.

(ii)



(iii) Bearing of F from $S = 244°$

13. (a) Interior angle $= \dfrac{(9-2)180°}{9} = 140°$

(b) (i) $BE = \sqrt{7^2 + 18^2 - 2(7)(18)\cos 115°}$

$= \sqrt{479.4998} = 21.9$ cm

(ii) $\dfrac{\sin D\hat{B}C}{11} = \dfrac{\sin 28°}{16} \Rightarrow D\hat{B}C = 18.8°$

(iii) $18 + 16 + 11 + DE + 7 = 62$

$\Rightarrow DE = 10$ cm

Area of $\Delta DEB = 109$ cm^2

$\Rightarrow \dfrac{1}{2}(21.9)(10)\sin D\hat{E}B = 109$

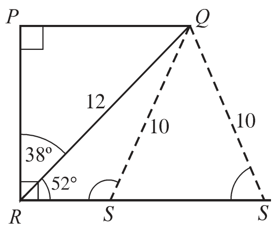
$\Rightarrow D\hat{E}B = 84.5°$

$\therefore$ Obtuse $D\hat{E}B = 180° - 84.5° = 95.5°$

14. (a) $PQ = 12\sin 38° = 7.39$ cm.

(b) $Q\hat{R}S = 90° - 38°$

$= 52°$



$\dfrac{\sin Q\hat{S}R}{12} = \dfrac{\sin 52°}{10}$

$\Rightarrow Q\hat{S}R = 71.0°$

or $180° - 71° = 109°$

15. (a) Draw a line BH from B to ED, such that BH is parallel to CD. Now in ΔEBH,

$HB = DC = \sqrt{(15.1)^2 - (2)^2} = 14.97$ m.

(b) $\cos E\hat{A}B = \dfrac{9^2 + 11^2 - 15.1^2}{2(9)(11)} \Rightarrow E\hat{A}B = 97.5°$