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MATHEMATICS
(Paper 2 - All Variants)

(Syllabus 4024)

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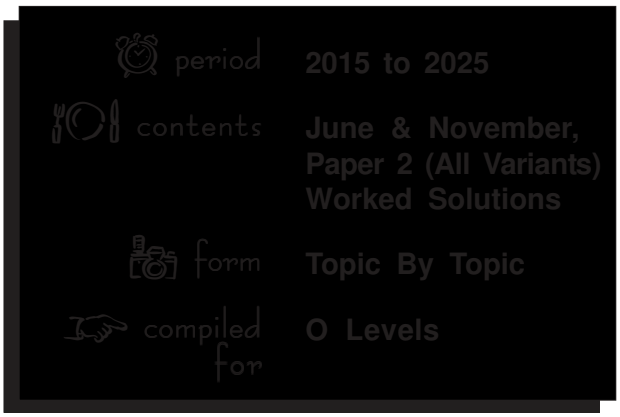
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Revision

June **2025** Paper 2 (All Variants)

November **2025** Paper 2 (All Variants)

TOPIC 9

Solutions of Equations

1. (a) Expand the brackets and simplify $(x-1)(x^2+x+1)$.

Answer ..... [2]

(b) Solve the equation $\dfrac{3x}{x+2}-\dfrac{4}{x-2}=3$.

Answer ..... [3]

(c) Solve these simultaneous equations.

$$4x-3y=4$$
$$4y-3x=-6.5$$

Answer x = ...

y = ... [4]

[June/2015/P21/Q6]

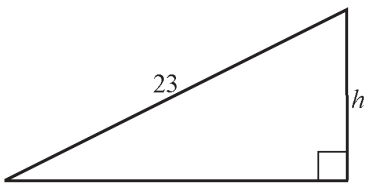
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2. (a) Solve $\dfrac{7}{3-2m}=4$.

Answer ..... [2]

(b) A right-angled triangle has a base that is 7 cm longer than its height, h cm. The hypotenuse of the triangle is 23 cm.

(i) Show that h satisfies the equation $h^2+7h-240=0$.



[2]

(ii) Write down an expression, in terms of h, for the area of the triangle.

Answer ..... cm^2 [1]

(iii) Hence state the exact area of the triangle.

Answer ..... cm^2 [1]

(iv) Solve $h^2 + 7h - 240 = 0$, giving your answers correct to 1 decimal place.

Answer h = ... or ..... [3]

(v) Calculate the perimeter of the triangle.

Answer ..... cm [1]

[June/2015/P22/Q9(b,c)]

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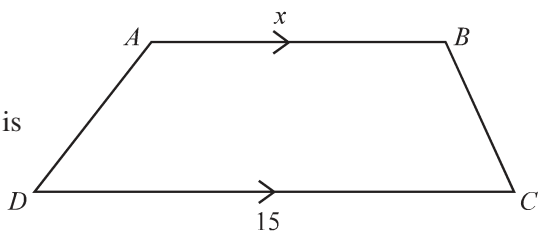
3. Solve $\frac{4}{x} + \frac{2}{x+2} = 3$.

Answer x = ... or ..... [3]

[Nov/2015/P21/Q5(b)]

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4. (a) $ABCD$ is a trapezium with AB parallel to DC.
DC = 15 cm and $AB = x$ cm.
The perpendicular distance between AB and DC is 3 cm less than the length of AB.
The area of $ABCD$ is 75 cm^2.



(i) Show that $x^2 + 12x - 195 = 0$.

[2]

(ii) Find AB, giving your answer correct to 1 decimal place.

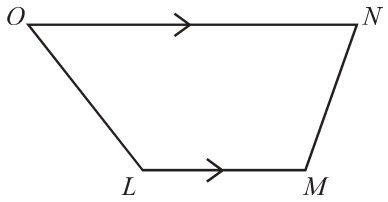
Answer ..... cm [3]

(iii) *AD* is 0.8 cm longer than *BC*.

Given that the perimeter of the trapezium is 38.0 cm, calculate *AD*.

Answer ..... cm [2]

(b) Another trapezium, *LMNO*, has *LM* parallel to *ON*. The reflex angle *LMN* = 252°.



(i) Calculate $M\widehat{N}O$.

Answer ..... [2]

(ii) The ratios of the angles inside the trapezium are $L\widehat{O}N : L\widehat{M}N = 1:2$ and $O\widehat{L}M : M\widehat{N}O = 1:k$.

Find *k*, giving your answer as a fraction in its simplest form.

Answer ..... [3]

[Nov/2015/P21/Q10]

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5. (a) (i) Solve the equation $\left(x+\frac{7}{2}\right)=\pm\frac{\sqrt{5}}{2}$.

Give both answers correct to 2 decimal places.

Answer x = ... or ..... [2]

(ii) The solutions of $\left(x+\frac{7}{2}\right)=\pm\frac{\sqrt{5}}{2}$ are also the solutions of $x^2+Bx+C=0$,

where *B* and *C* are integers.

Find *B* and *C*.

Answer B = ... C = ... [3]

(b) Solve the inequality $7-3x>13$.

Answer x ..... [2]

(c) Factorise $6x-3yt+18y-xt$.

Answer ..... [2]

(d) Solve these simultaneous equations.

$$3a+4b=-13$$
$$5a+6b=-11$$

Answer a = ...

b = ... [4]

[Nov/2015/P22/Q6]

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6. (a) Solve the equation $\frac{p-1}{7-p}=5$.

Answer ..... [2]

(b) (i) Factorise $4t^2+35t-9$.

Answer ..... [2]

(ii) Hence solve the equation $4t^2+35t-9=0$.

Answer ..... [1]

[June/2016/P21/Q2(a,d)]

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7. (a) $p=\frac{8-5q}{q}$

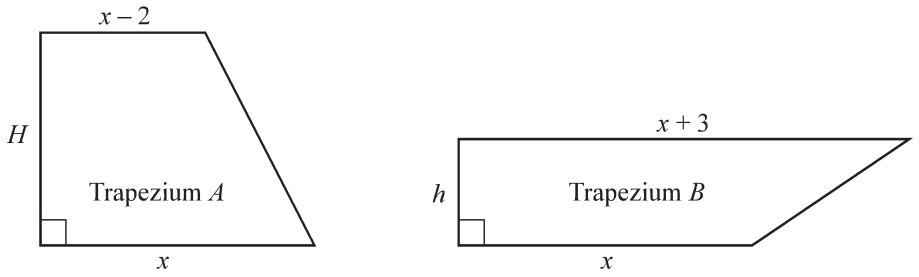
(i) Find p when $q = 2.6$.

Answer ..... [1]

(ii) Express q in terms of p.

Answer ..... [2]

(b)



The lengths of the parallel sides of trapezium A are x cm and $(x - 2)$ cm.
The lengths of the parallel sides of trapezium B are x cm and $(x + 3)$ cm.
The height of trapezium A is H cm and the height of trapezium B is h cm.
The area of each trapezium is 15 cm^2.

(i) Show that $H = \dfrac{15}{x-1}$ and $h = \dfrac{30}{2x+3}$.

[2]

(ii) Find an expression in terms of x for the difference in height, $H - h$, between trapezium A and trapezium B, and show that it simplifies to $\dfrac{75}{(x-1)(2x+3)}$.

[3]

(iii) The difference in height is 1.5 cm.

(a) Show that $2x^2 + x - 53 = 0$.

[2]

(b) Find x, giving your answer correct to 2 decimal places.

Answer x = ... [2]

[June/2016/P21/Q8]

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8. (a) Factorise fully $8x^2y - 12x^5$.

Answer ..... [1]

(b) Solve $4x-2(x+5)=3$.

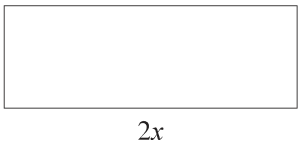
Answer ..... [2]

(c) Solve $7-5y<20$.

Answer y ..... [2]

(d) A rectangle has length $2x$ cm, perimeter 18 cm and area 10 cm^2.

(i) Show that $2x^2-9x+5=0$.



[2]

(ii) Solve $2x^2-9x+5=0$, giving your answers correct to 2 decimal places.

Answer x = ... or ..... [3]

(iii) Find the difference between the length and the width of the rectangle.

Answer ..... cm [1]

[June/2016/P22/Q5]

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9. On Monday, Abdul sold 140 boxes of matches at 30 cents per box.

(a) Calculate the income, in dollars, Abdul received on Monday.

Answer $ ..... [1]

(b) On Tuesday, the price per box decreased by 10% and the number of boxes sold increased by 30%.

Calculate the percentage change in the income.

Answer ..... % [3]

(c) On Wednesday, the price of a box was y cents less than it was on Monday.
Abdul sold $4y$ more boxes on Wednesday than he did on Monday.

(i) Write down an expression, in terms of y, for the income received on Wednesday.
Give your answer in dollars.

Answer $ ..... [2]

(ii) Given that this income is equal to \$40, write down an equation in y and show that it simplifies to

$$y^2 + 5y - 50 = 0.$$

[2]

(iii) Solve the equation $y^2 + 5y - 50 = 0$.

Answer y = ... or ..... [3]

(iv) Hence find the number of boxes sold on Wednesday.

Answer ..... [1]

[Nov/2016/P21/Q9]

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10. (i) Find the two solutions of $5x - 1 = \pm 9$.

Answer x = ... or ..... [2]

(ii) The solutions of $5x - 1 = \pm 9$ are also the solutions of $5x^2 + Bx + C = 0$, where B and C are integers. Find B and C.

Answer B = ... , C = ... [2]

[Nov/2016/P22/Q2(e)]

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11. (a) $x = \sqrt{a^2 + b^2}$

(i) Calculate x when $a = -0.73$ and $b = 1.84$.

Answer ..... [1]

(ii) Express b in terms of x and a.

Answer b = ... [2]

ANSWERS

Topic 9 - Solutions of Equations

1. **(a)** $(x-1)(x^2+x+1)$
$= x^3 + x^2 + x - x^2 - x - 1 = x^3 - 1$

(b) $\dfrac{3x}{x+2} - \dfrac{4}{x-2} = 3$
$\Rightarrow \dfrac{3x(x-2) - 4(x+2)}{(x+2)(x-2)} = 3$
$\Rightarrow 3x^2 - 10x - 8 = 3(x^2 - 4) \Rightarrow x = \dfrac{2}{5}$

(c) $4x - 3y = 4$ .....(1), $3x - 4y = 6.5$ .....(2)
$(1)\times 3$: $12x - 9y = 12$ .....(3)
$(2)\times 4$: $12x - 16y = 26$ .....(4)
$(3) - (4)$ gives, $y = -2$
Subst. $y = -2$ into (1) to get, $x = -0.5$

2. **(a)** $\dfrac{7}{3-2m} = 4 \Rightarrow 7 = 12 - 8m \Rightarrow m = \dfrac{5}{8}$

(b) **(i)** By Pythagoras, $(7+h)^2 + h^2 = (23)^2$
$\Rightarrow (49 + 14h + h^2) + h^2 = 529$
$\Rightarrow 2h^2 + 14h - 480 = 0$
$\Rightarrow h^2 + 7h - 240 = 0$

(ii) Area $= \dfrac{1}{2}(7+h)(h) = \dfrac{1}{2}(7h + h^2)$ cm^2

(iii) From (b) (i), $h^2 + 7h = 240$
$\therefore$ Area of $\Delta = \dfrac{1}{2}(240) = 120$ cm^2

(iv) $h = \dfrac{-7 \pm \sqrt{(7)^2 - 4(1)(-240)}}{2(1)}$
$= \dfrac{-7 \pm \sqrt{1009}}{2}$
$\therefore \quad h = 12.4$ or -19.4

(v) Perimeter $= h + h + 7 + 23$
$= 12.38 + 12.38 + 7 + 23$
$= 54.76 \approx 54.8$ cm

3. $\dfrac{4}{x} + \dfrac{2}{x+2} = 3$
$\Rightarrow \dfrac{4(x+2) + 2x}{x(x+2)} = 3 \Rightarrow 6x + 8 = 3(x^2 + 2x)$
$\Rightarrow 3x^2 = 8 \Rightarrow x = \pm\sqrt{\dfrac{8}{3}}$

4. **(a)** **(i)** Area of trapezium $= 75$ cm^2
$\Rightarrow \dfrac{1}{2}(x+15)(x-3) = 75$
$\Rightarrow x^2 - 3x + 15x - 45 = 150$
$\Rightarrow x^2 + 12x - 195 = 0$

(ii) $x = \dfrac{-12 \pm \sqrt{(12)^2 - 4(1)(-195)}}{2(1)}$
$= \dfrac{-12 \pm \sqrt{924}}{2}$
$\therefore \quad x = 9.2$ or -21.2, so $AB = 9.2$ cm

(iii) Let $BC = y$ cm
$\therefore \quad 9.2 + y + 15 + (y + 0.8) = 38$
$\Rightarrow 2y + 25 = 38 \Rightarrow y = 6.5$
$\therefore \quad AD = 6.5 + 0.8 = 7.3$ cm

(b) **(i)** Obtuse $L\hat{M}N = 360° - 252° = 108°$
$\therefore \quad M\hat{N}O = 180° - 108° = 72°$

(ii) $L\hat{O}N = \dfrac{1}{2}(108°) = 54°$
$O\hat{L}M = 360° - (108° + 72° + 54°)$
$= 126°$
Given, $\dfrac{O\hat{L}M}{M\hat{N}O} = \dfrac{1}{k}$
$\Rightarrow \dfrac{126°}{72°} = \dfrac{1}{k} \Rightarrow k = \dfrac{72°}{126°} = \dfrac{4}{7}$

5. **(a)** **(i)** $\left(x + \dfrac{7}{2}\right) = \pm\dfrac{\sqrt{5}}{2}$
$\Rightarrow x + \dfrac{7}{2} = \dfrac{\sqrt{5}}{2}$ or $x + \dfrac{7}{2} = -\dfrac{\sqrt{5}}{2}$
$\therefore \quad x = -2.38$ or -4.62

(ii) $\left(x+\frac{7}{2}\right)^2=\left(\pm\frac{\sqrt{5}}{2}\right)^2$

$\Rightarrow\ x^2+7x+\frac{49}{4}=\frac{5}{4}$

$\Rightarrow\ x^2+7x+11=0$

$\therefore\ B=7,\ C=11$

(b) $7-3x>13\ \Rightarrow\ -3x>6\ \Rightarrow\ x<-2$

(c) Re-arrange as, $6x-xt-3yt+18y$

$=x(6-t)-3y(t-6)$

$=x(6-t)+3y(6-t)=(6-t)(x+3y)$

(d) $3a+4b=-13$.....(1), $5a+6b=-11$....(2)

$(1)\times 3$: $9a+12b=-39$ .....(3)

$(2)\times 2$: $10a+12b=-22$ .....(4)

$(3)-(4)$ gives, $a=17$

Subst. $a=17$ into (1) to get, $b=-16$

6. (a) $\frac{p-1}{7-p}=5\ \Rightarrow\ p-1=35-5p\ \Rightarrow\ p=6$

(b) (i) $4t^2+35t-9=(4t-1)(t+9)$

(ii) Using (b) (i), $(4t-1)(t+9)=0$

$\Rightarrow\ t=\frac{1}{4}$ or $t=-9$

7. (a) (i) $p=\frac{8-5(2.6)}{2.6}=-1.92$

(ii) $p=\frac{8-5q}{q}$

$\Rightarrow\ qp=8-5q\ \Rightarrow\ q=\frac{8}{p+5}$

(b) (i) Area of trapezium $A=15\ \text{cm}^2$

$\Rightarrow\ \frac{1}{2}H(x+x-2)=15\ \Rightarrow\ H=\frac{15}{x-1}$

For trap. B, $\frac{1}{2}h(x+x+3)=15$

$\Rightarrow\ h(2x+3)=30\ \Rightarrow\ h=\frac{30}{(2x+3)}$

(ii) $H-h=\frac{15}{x-1}-\frac{30}{(2x+3)}$

$=\frac{15(2x+3)-30(x-1)}{(x-1)(2x+3)}$

$=\frac{75}{(x-1)(2x+3)}$

(iii) (a) $\frac{75}{(x-1)(2x+3)}=1.5$

$\Rightarrow\ 75=1.5(2x^2+x-3)$

$\Rightarrow\ 2x^2+x-53=0$

(b) $x=\frac{-1\pm\sqrt{(1)^2-4(2)(-53)}}{2(2)}$

$=\frac{-1\pm\sqrt{425}}{4}$, $\therefore\ x=4.90$

8. (a) $8x^2y-12x^5=4x^2(2y-3x^3)$

(b) $4x-2(x+5)=3$

$\Rightarrow\ 4x-2x-10=3\ \Rightarrow\ x=6.5$

(c) $7-5y<20\ \Rightarrow\ -5y<13\ \Rightarrow\ y>-2.6$

(d) (i) Let width of the rectangle be w,

Given perimeter $=18$ cm

$\Rightarrow\ 2(2x+w)=18\ \Rightarrow\ w=9-2x$

Area, $2x\times w=10$

$\Rightarrow\ 2x\times(9-2x)=10$

$\Rightarrow\ 2x^2-9x+5=0$

(ii) $x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(5)}}{2(2)}$

$=\frac{9\pm\sqrt{41}}{4}$, $\therefore\ x=3.85$ or 0.65

(iii) Taking $x=3.85$,

Length $=2(3.85)=7.7$ cm

width $=9-2(3.85)=1.3$ cm

Difference $=7.7-1.3=6.4$ cm.

9. (a) $\$0.30\times 140=\$42$

(b) New price $=90\%\times 0.30=\$0.27$

No. of boxes sold $=130\%\times 140=182$

% change $=\frac{(0.27\times 182)-42}{42}\times 100=17\%$

(c) (i) Income $=\$\ \frac{(30-y)(140+4y)}{100}$

(ii) $\frac{(30-y)(140+4y)}{100}=40$

$\Rightarrow\ 4200-20y-4y^2=4000$

$\Rightarrow\ y^2+5y-50=0$

(iii) $y^2+5y-50=0$

$\Rightarrow\ (y-5)(y+10)=0$

$\Rightarrow\ y=5$ or $y=-10$

(iv) No. of boxes sold $=140+4(5)=160$

10. **(i)** Either, $5x-1=9$ or $5x-1=-9$

$\Rightarrow x=2$ or $-\frac{8}{5}$

(ii) $(5x-1)^2=81$

$\Rightarrow 25x^2-10x-80=0$ (divide by 5)

$\Rightarrow 5x^2-2x-16=0$

$\therefore B=-2,\quad C=-16$

11. **(a)** **(i)** $x=\sqrt{(-0.73)^2+(1.84)^2}=1.98$

(ii) $x=\sqrt{a^2+b^2}\ \Rightarrow\ b^2=x^2-a^2$

$\Rightarrow b=\sqrt{(x+a)(x-a)}$

(b) **(i)** $PQ=\frac{17}{x+5}$ cm

(ii) $AB=(\frac{17}{x+5}+3)$ cm

Area of $ABCD=17$

$\Rightarrow x\times(\frac{17}{x+5}+3)=17$

$\Rightarrow \frac{17x+3x(x+5)}{x+5}=17$

Simplify to get, $3x^2+15x-85=0$

(iii) $x=\frac{-15\pm\sqrt{(15)^2-4(3)(-85)}}{2(3)}$

$=\frac{-15\pm\sqrt{1245}}{6}$

$\therefore\ x=3.38$ or -8.38

(iv) $PQ=\frac{17}{3.38+5}=2.03$ cm

$PS=3.38+5=8.38$ cm

Perimeter $=2(2.03+8.38)=20.82$

12. **(i)** $4x^2+12x+9=(2x+3)^2$

(ii) $4x^2+12x+9=49$

$\Rightarrow (2x+3)^2=49\ \Rightarrow\ (2x+3)=\pm 7$

$\therefore\ x=2$ or -5

13. $3x^2-x-5=0$. By quadratic formula,

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-5)}}{2(3)}=\frac{1\pm\sqrt{61}}{6}$

$\therefore\ x=1.47$ or -1.14

14. **(a)** $\frac{y}{2y+3}=\frac{2}{y+5}$

$\Rightarrow y(y+5)=2(2y+3)$

$\Rightarrow y^2+y-6=0\ \Rightarrow\ (y+3)(y-2)=0$

$\therefore\ y=-3$ or 2

(b) $p=\frac{4t+1}{2-t}$

$\Rightarrow 2p-pt=4t+1$

$\Rightarrow 4t+pt=2p-1\ \Rightarrow\ t=\frac{2p-1}{4+p}$

15. **(a)** 3000 litres —— 12 minutes

1750 litres —— $\frac{12}{3000}\times 1750=7$ minutes

(b) **(i)** $\frac{2500}{x}$ minutes

(ii) Time for large pump $=\frac{2500}{x+20}$ minutes

$\frac{2500}{x}-\frac{2500}{x+20}=15$

$\Rightarrow 2500\left(\frac{x+20-x}{x(x+20)}\right)=15$

$\Rightarrow 50000=15x(x+20)$

Simplify to, $3x^2+60x-10000=0$

(iii) $x=\frac{-60\pm\sqrt{(60)^2-4(3)(-10000)}}{2(3)}$

$=\frac{-60\pm\sqrt{123600}}{6}$

$\therefore\ x=48.59$ or -68.59

(iv) Time for larger pump $=\frac{2500}{48.59+20}$

$=36.448$ minutes

$=36$ minutes 27 seconds

16. **(a)** $2x(x+1)=3(4-x)$

$\Rightarrow 2x^2+5x-12=0$

$\Rightarrow (2x-3)(x+4)=0\ \ \therefore\ x=\frac{3}{2}$ or -4

(b) **(i)** Let p be cost of a pen and n be the cost of a notebook.

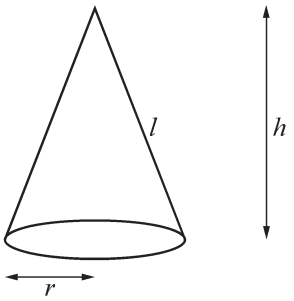
$\therefore\ 3p+2n=4.8\ \ \&\ \ 5p+4n=9$

TOPIC 14

Graphs of Functions

1. [Curved surface area of a cone = πrl]

The diagram shows a solid cone with radius r cm, height h cm and slant height l cm.
Suleman makes some solid cones. The slant height of each of his cones is 4 cm more than its radius.
Use $\pi = 3$ throughout this question.



(a) Show that the **total** surface area, A cm^2, of each of Suleman's cones is given by $A = 6r(r + 2)$.

[2]

(b) Complete the table for $A = 6r(r + 2)$.

r	0	1	2	3	4	5	6
A	0	18			144	210	288

[1]

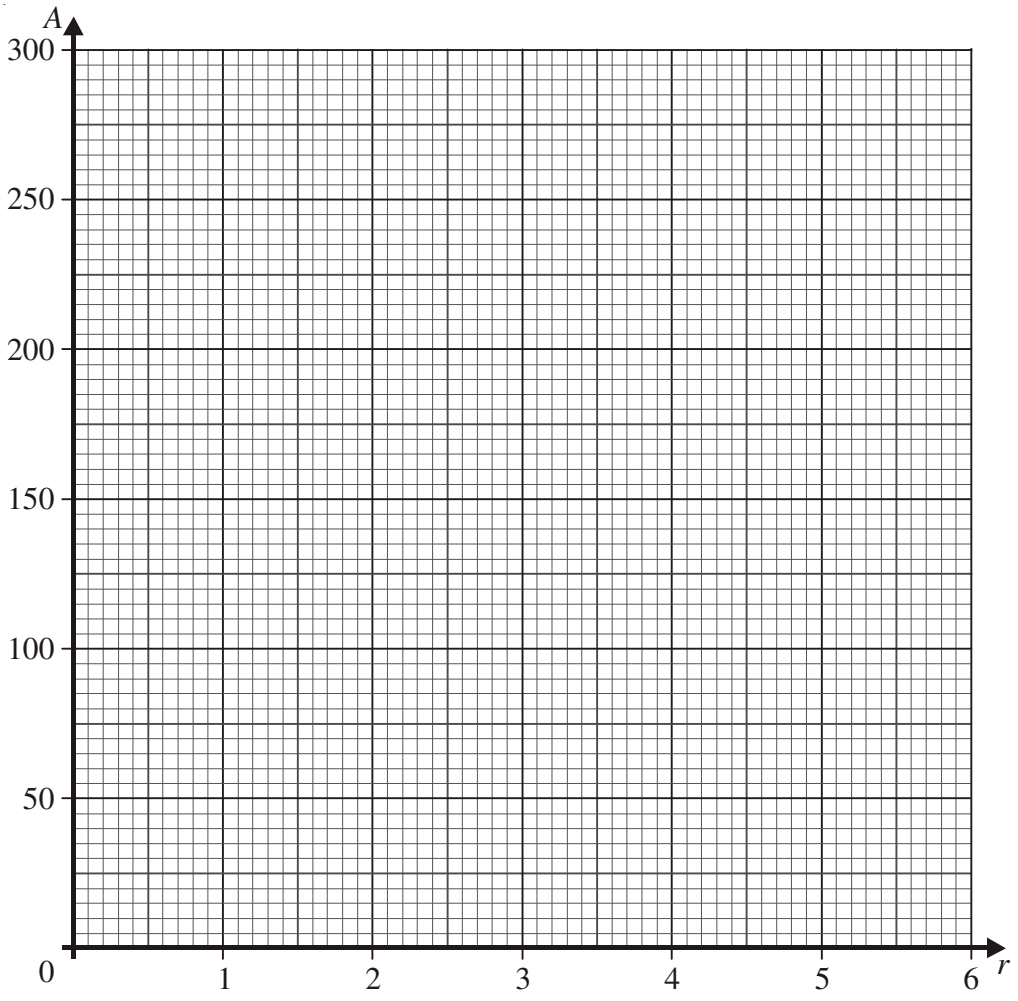
(c) On the grid below, draw the graph of $A = 6r(r + 2)$. [2]

(d) Find an expression for h in terms of r.

Answer h = ... [2]

(e) The height of one of Suleman's cones is 12 cm.
Calculate its radius.

Answer ..... cm [2]

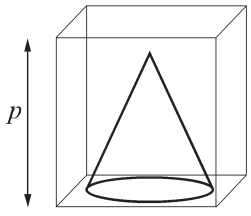


(f) Another of Suleman's cones has a surface area of 200 cm^2.

(i) Use your graph to find the radius of this cone.

Answer ..... cm [1]

(ii) This cone is placed in a box of height p cm, where p is an integer.
Find the smallest possible value of p.



Answer p = ... [2]

[Nov/2015/P21/Q8]

2. The distance, d metres, of a moving object from an observer after t minutes is given by

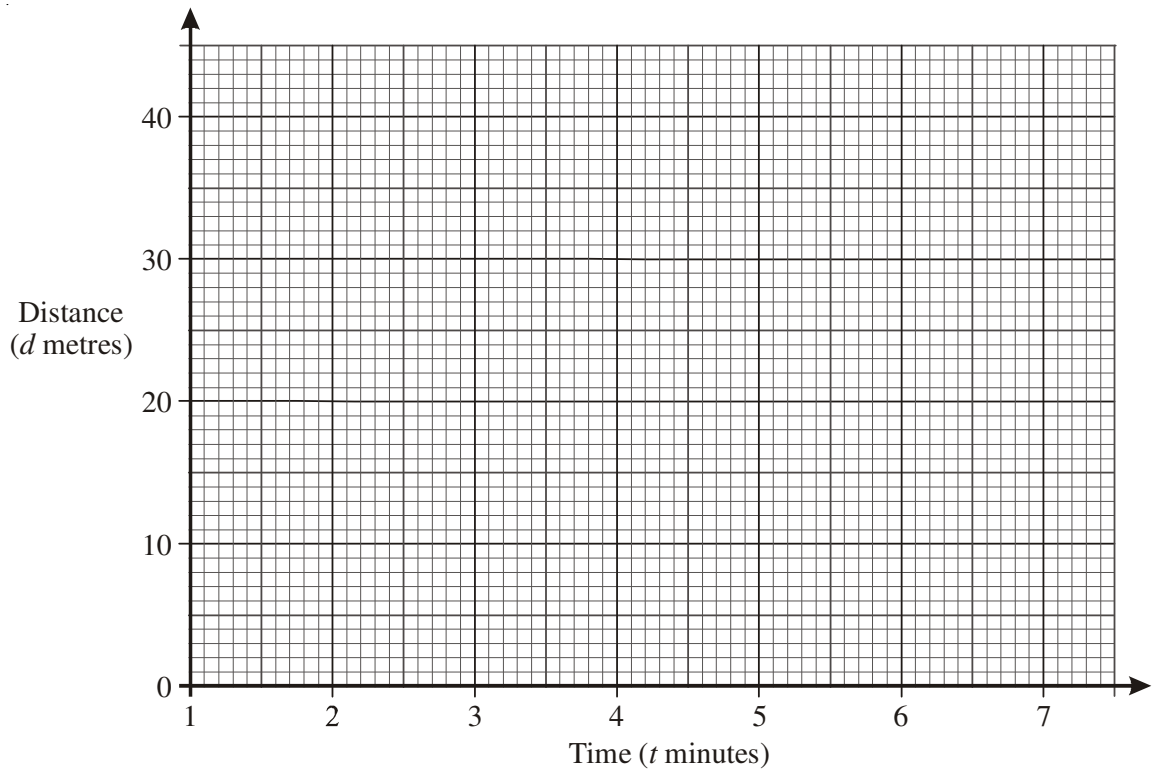
$$d = t^2 + \frac{48}{t} - 20.$$

(a) Some values of t and d are given in the table.
The values of d are given to the nearest whole number where appropriate.

t	1	1.5	2	2.5	3	3.5	4	4.5	5	6	7
d	29	14	8	5	5	6	8	11	15	24	

Complete the table. [1]

(b) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

(c) (i) By drawing a tangent, calculate the gradient of the curve when $t = 4$.

Answer ..... [2]

(ii) Explain what this gradient represents.

Answer ..... [1]

(d) For how long is the object less than 10 metres from the observer?

Answer ..... minutes [2]

(e) (i) Using your graph, write down the two values of t when the object is 12 metres from the observer.
For each value of t, state whether the object is moving towards or away from the observer.

Answer When t = ..., the object is moving ..... the observer.

When t = ..., the object is moving ..... the observer. [2]

(ii) Write down the equation that gives the values of t when the object is 12 metres from the observer.

Answer ..... [1]

(iii) This equation is equivalent to $t^3 + At + 48 = 0$.
Find A.

Answer A = ... [1]

[Nov/2015/P22/Q9]

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3. The table below is for $y = x^2 + x - 3$.

x	−3	−2	−1	0	1	2
y	3	−1	−3	−3	−1	3

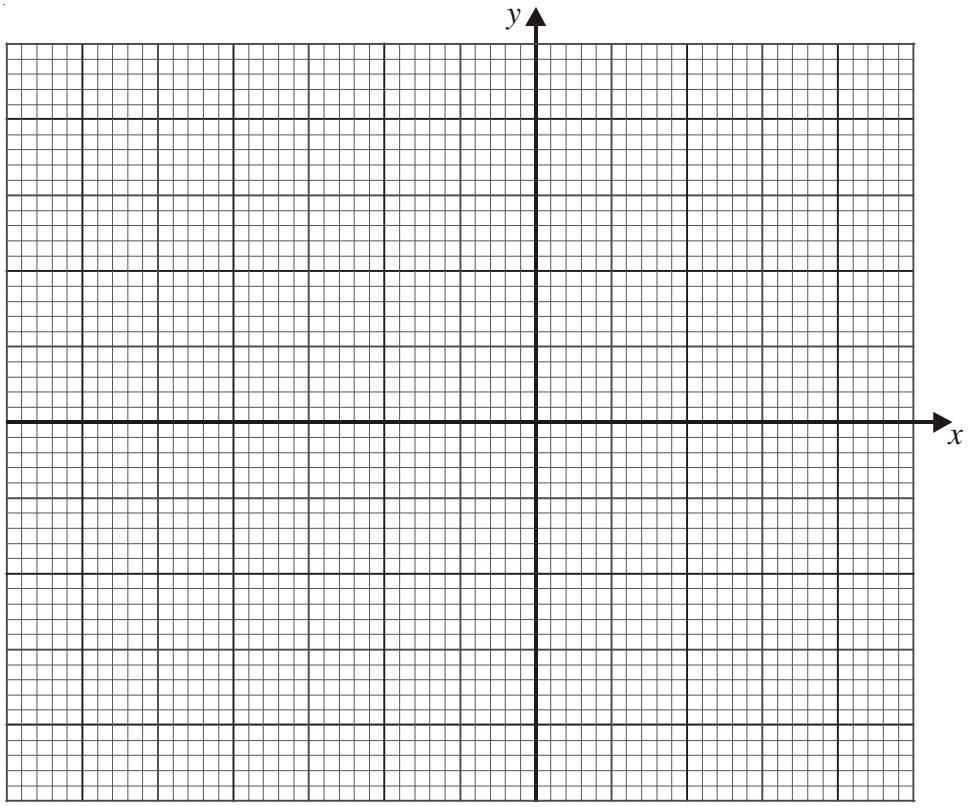
(a) Using a scale of 2 cm to 1 unit on the x-axis for $-3 \le x \le 2$
and a scale of 1 cm to 1 unit on the y-axis for $-4 \le y \le 4$,
plot the points from the table and join them with a smooth curve. [2]

(b) (i) Use your graph to estimate the solutions of the equation $x^2 + x - 3 = 0$.

Answer x = ... or ..... [1]

(ii) Use your graph to estimate the solutions of the equation $x^2 + x - 5 = 0$.

Answer x = ... or ..... [2]



(c) By drawing a tangent, estimate the gradient of the curve at (1, –1).

Answer ..... [2]

(d) The equation $x^2 - x - 1 = 0$ can be solved by drawing a straight line on the graph of $y = x^2 + x - 3$.

(i) Find the equation of this straight line.

Answer ..... [2]

(ii) Draw this straight line and hence solve $x^2 - x - 1 = 0$.

Answer x = ... or ..... [2]

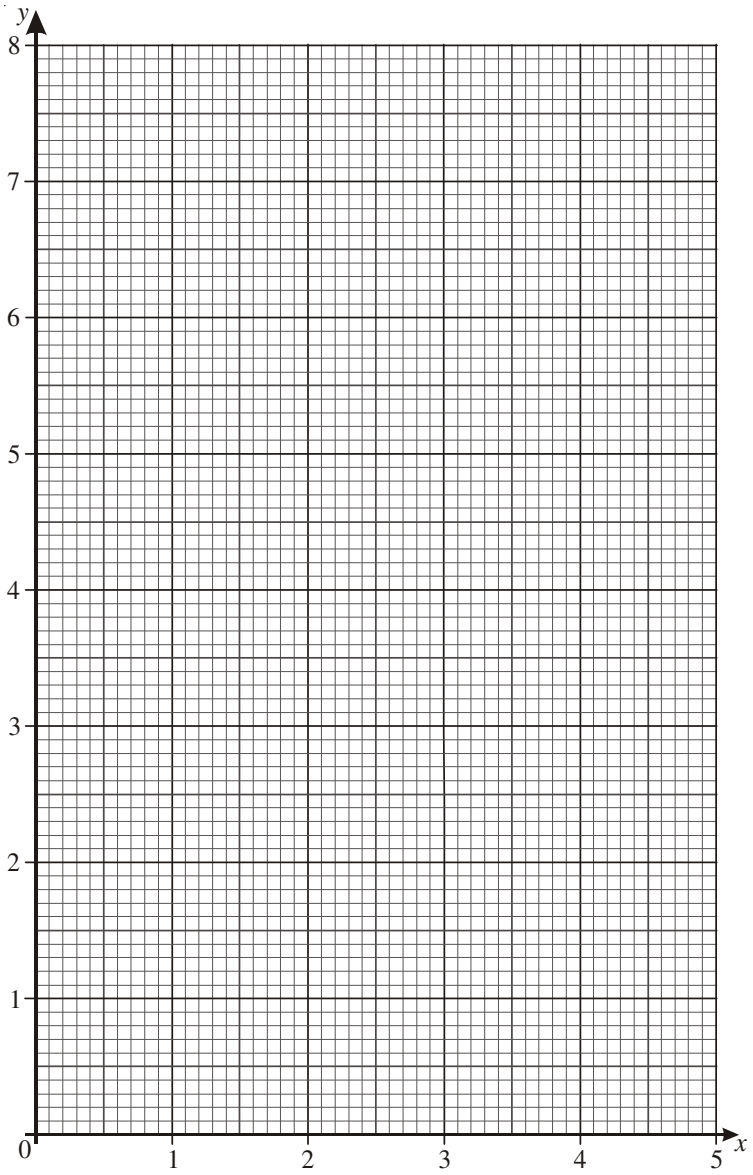
[June/2016/P21/Q3]

4. The table below shows some values of x and the corresponding values of y for $y = \frac{1}{4} \times 2^x$.

x	0	1	2	3	4	5
y	$\frac{1}{4}$		1	2	4	8

(a) Complete the table. [1]

(b) On the grid below, draw the graph of $y = \frac{1}{4} \times 2^x$. [2]



(c) By drawing a suitable line, find the gradient of your graph where $x = 4$.

Answer ..... [2]

(d) (i) Show that the line $2x + y = 6$, together with the graph of $y = \frac{1}{4} \times 2^x$, can be used to solve the equation $2^x + 8x - 24 = 0$.

[1]

(ii) Hence solve $2^x + 8x - 24 = 0$.

Answer $x =$ ..... [2]

(e) The points P and Q are (2, 3) and (5, 4) respectively.

(i) Find the gradient of PQ.

Answer ..... [1]

(ii) On the grid, draw the line l, parallel to PQ, that touches the curve $y = \frac{1}{4} \times 2^x$. [1]

(iii) Write down the equation of l.

Answer ..... [2]

[June/2016/P22/Q8]

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5. $$y = \frac{3}{5} \times 2^x$$

The table shows some values of x and the corresponding values of y, correct to one decimal place where necessary.

x	−1.5	−1	0	1	2	2.5	3	3.5	4
y	p	0.3	0.6	1.2	2.4	3.4	4.8	6.8	9.6

(a) Calculate p.

Answer ..... [1]

(b) On the grid,

- using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \le x \le 4$,
- using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \le y \le 10$,
- plot the points from the table and join them with a smooth curve. [3]

ANSWERS

Topic 14 - Graphs of Functions

1. **(a)** $A = \pi r^2 + \pi rl$

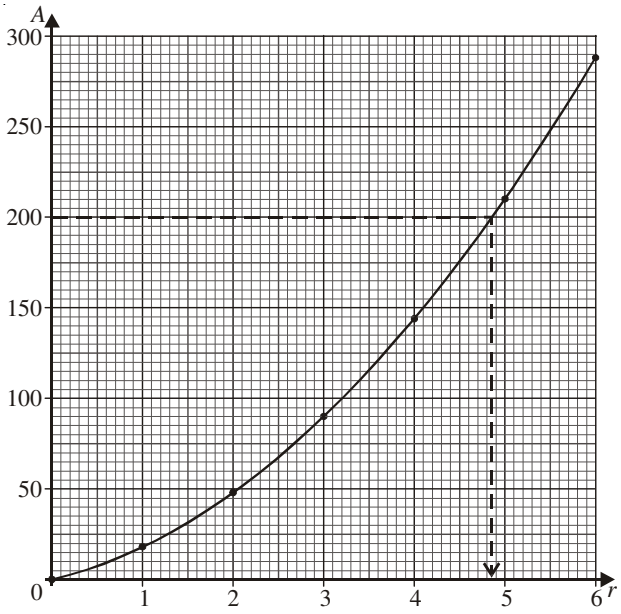
$\Rightarrow A = 3r^2 + 3r(r+4)$

$\Rightarrow A = 6r^2 + 12r \quad \Rightarrow \quad A = 6r(r+2).$

(b) When $r = 2$, $A = 6(2)(2+2) = 48$

When $r = 3$, $A = 6(3)(3+2) = 90$

(c)



(d) Using Pythagoras, $h = \sqrt{(r+4)^2 - r^2}$

$\Rightarrow h = \sqrt{r^2 + 8r + 16 - r^2}$

$\Rightarrow h = \sqrt{8r + 16}$

(e) $\sqrt{8r+16} = 12$

$\Rightarrow 8r + 16 = 144 \quad \Rightarrow \quad r = 16$ cm

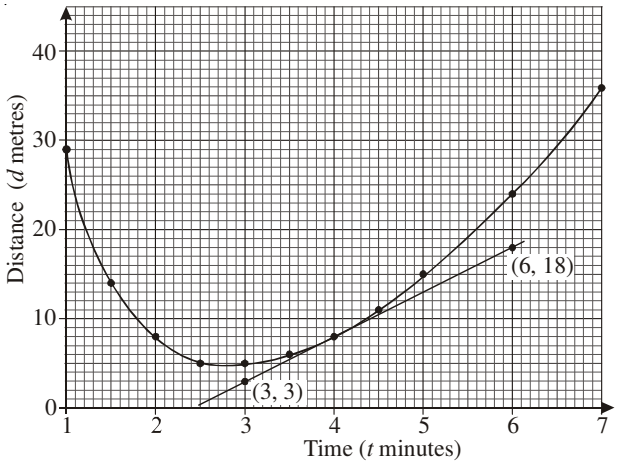
(f) **(i)** $r = 4.85$ cm

(ii) $h = \sqrt{8(4.85)+16} = 7.40$

$\therefore$ smallest value of $p = 8$ cm

2. **(a)** When $t = 7$, $d = (7)^2 + \frac{48}{7} - 20 \approx 36$

(b)



(c) **(i)** Using (3, 3) and (6, 18) on tangent,

$\text{gradient} = \frac{18-3}{6-3} = 5$

(ii) The gradient represents the speed of the object at 4 minutes.

(d) At $d = 10$ m, $t = 1.8$ & 4.4 min.

Length of time $= 4.4 - 1.8 = 2.6$ min.

(e) **(i)** When $t =$ ..**1.63**.., the object is moving ..**towards**.... the observer.

When $t =$ .....**4.65**....., the object is moving ..**away from** the observer.

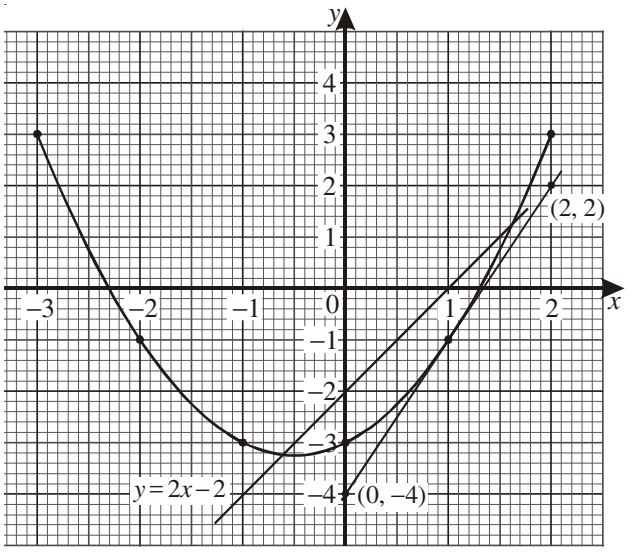
(ii) $t^2 + \frac{48}{t} - 20 = 12$

$\Rightarrow t^2 + \frac{48}{t} - 32 = 0$

(iii) $t^2 + \frac{48}{t} - 32 = 0$

$\Rightarrow t^3 - 32t + 48 = 0, \quad \therefore \quad A = -32$

3. (a)



(b) (i) $x^2 + x - 3 = 0 \Rightarrow y = 0$

$\therefore$ from graph, $x = -2.3$ or 1.3

(ii) $x^2 + x - 5 = 0$

$\Rightarrow x^2 + x - 3 - 2 = 0 \Rightarrow y = 2$

$\therefore$ from graph, $x = -2.8$ or 1.8

(c) Using $(0, -4)$ & $(2, 2)$ on the tangent,

gradient $= \dfrac{2-(-4)}{2-0} = 3$

(d) (i) $x^2 - x - 1 = 0$

$\Rightarrow x^2 - x - 1 + 2x - 2 = 2x - 2$

$\Rightarrow x^2 + x - 3 = 2x - 2$

$\Rightarrow y = 2x - 2$

(ii) Refer to graph for $y = 2x - 2$.

The graphs meet at, $x = -0.6$, or 1.6

4. (a) When $x = 1$, $y = \dfrac{1}{4} \times 2^1 = \dfrac{1}{2}$

(b) Refer to graph.

(c) Taking (3.3, 2) and (4.6, 5.7) on the tangent, gradient $= \dfrac{5.7-2}{4.6-3.3} = 2.85$

(d) (i) Substitute eq. of line into curve,

$2x + (\dfrac{1}{4} \times 2^x) = 6$

$\Rightarrow \dfrac{1}{4} \times 2^x = 6 - 2x$

$\Rightarrow 2^x + 8x - 24 = 0$

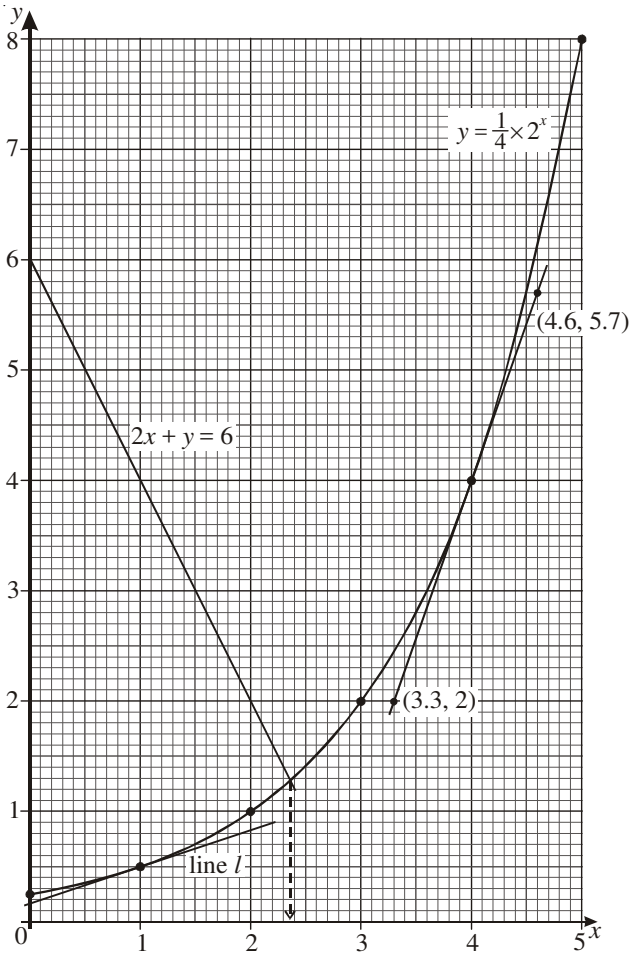
(ii) From graph, $x = 2.35$

(e) (i) Gradient of $PQ = \dfrac{4-3}{5-2} = \dfrac{1}{3}$

(ii) Refer to graph.

(iii) From graph, y-intercept of $l = 0.17$

$\therefore$ equation of l: $y = \dfrac{1}{3}x + 0.17$



5. (a) $p = \dfrac{3}{5} \times 2^{-1.5} = 0.21$

(b) Refer to graph on next page.

(c) Taking (1.3, 0.6) and (3.6, 6) on the tangent, gradient $= \dfrac{6-0.6}{3.6-1.3} = 2.35$

(d) (i) Refer to graph.

(ii) Gradient $= \dfrac{3.6-0}{2+0.4} = 1.5$

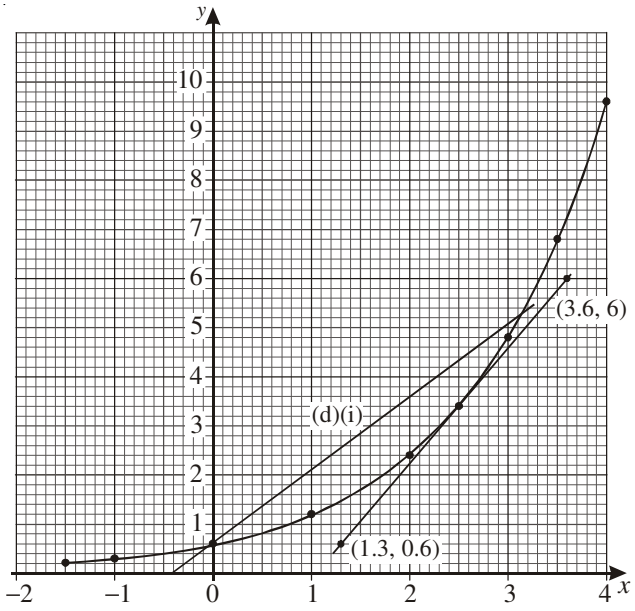
Equation, $y = 1.5x + 0.6$

(iii) $x = 0$ and $x = 3.15$

(iv) Subst. line into curve,

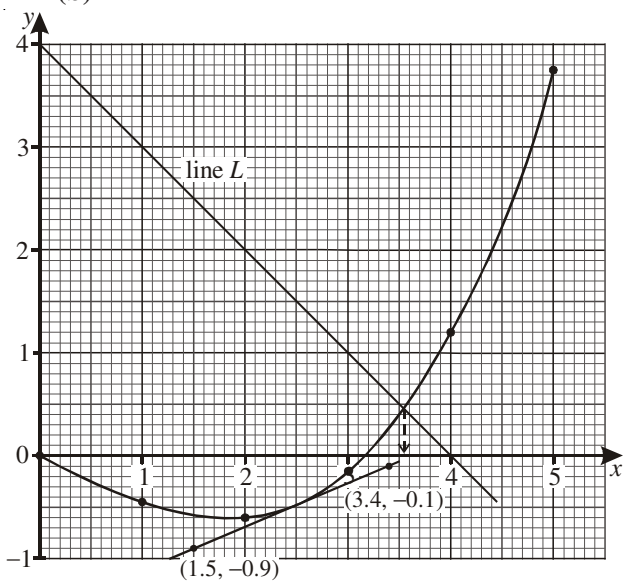
$\frac{3}{5}\times 2^x = 1.5x + 0.6$

$\Rightarrow\ 2^x = 2.5x + 1. \quad \therefore\ A = 2.5,\ B = 1$



6. **(a)** When $x = 5$, $y = \frac{5}{20}(5^2 - 10) = 3.75$

(b)



(c) Using $(1.5,\ -0.9)$ and $(3.4,\ -0.1)$,

$\text{gradient} = \frac{-0.1-(-0.9)}{3.4-1.5} = 0.421$

(d) $\frac{x}{20}(x^2 - 10) = 0 \quad \Rightarrow \quad y = 0$

From graph, at $y = 0$, $x = 3.17$

(e) **(i)** $x^3 + 10x - 80 = 0$

$\Rightarrow\ x^3 = 80 - 10x$

$\Rightarrow\ x^3 - 10x = 80 - 20x$

$\Rightarrow\ x(x^2 - 10) = 20(4 - x)$

$\Rightarrow\ \frac{x}{20}(x^2 - 10) = 4 - x$

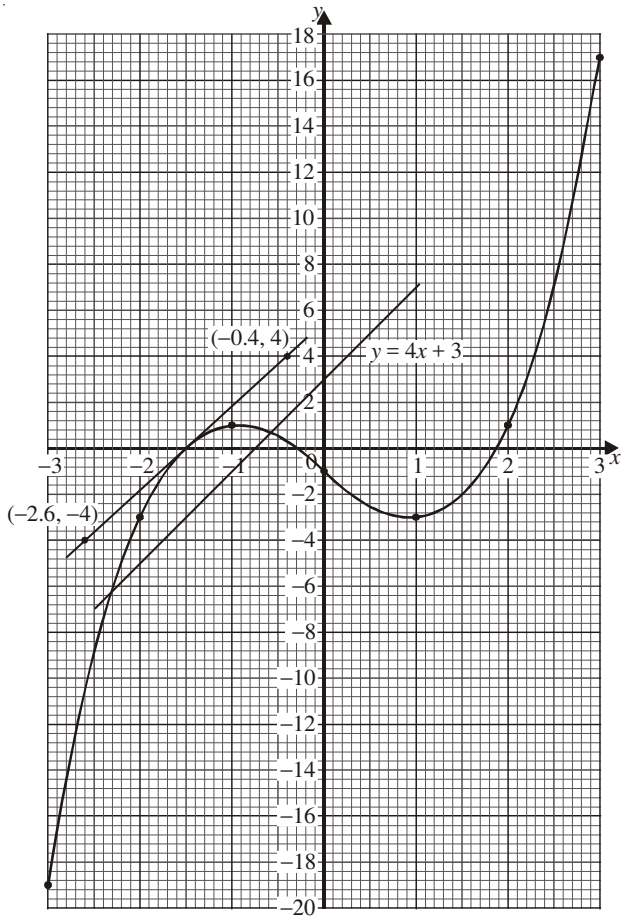
$\Rightarrow\ y = 4 - x$

(ii) Refer to graph on next page.

(iii) From graph, $x = 3.55$

7. **(a)** For $x = 3$, $y = 3^3 - 3(3) - 1 = 17$

(b)



(c) $x^3 - 3x - 1 = 0 \quad \Rightarrow \quad y = 0$

From graph, $x = -1.5,\ -0.3,\ 1.85$

(d) Using $(-2.6, -4)$ and $(-0.4, 4)$,

$\text{gradient} = \frac{4-(-4)}{-0.4-(-2.6)} = 3.64$

(e) **(i)** Refer to graph.

TOPIC 23

Bearings and Trigonometry

1. (a) (i) Evaluate $\dfrac{8\sin 54°}{\sin 18°}$.

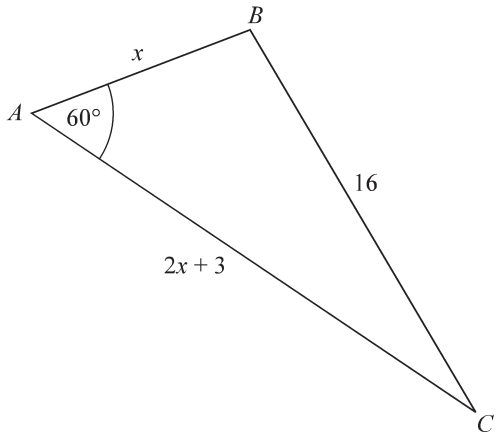
Answer ..... [1]

(ii) Evaluate $\sqrt{4.73^2 - 1.65\sin 43°}$.

Answer ..... [1]

(b) In the triangle ABC, $BC = 16$ cm and $B\hat{A}C = 60°$.
$AB = x$ cm and $AC = 2x + 3$ cm.

(i) Form an equation in x and show that it simplifies to $3x^2 + 9x - 247 = 0$.



[4]

(ii) Solve the equation $3x^2 + 9x - 247 = 0$, giving your answers correct to 2 decimal places.

Answer x = ... or ..... [3]

(iii) Hence write down the lengths of AB and AC.

Answer AB = ... cm AC = ... cm [1]

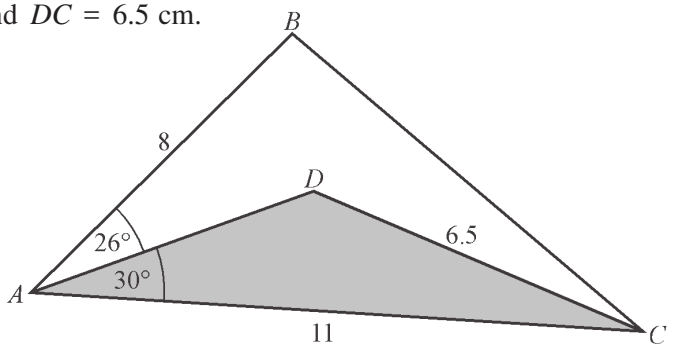
(iv) Find the area of triangle ABC.

Answer ..... cm^2 [2]

[June/2015/P21/Q7]

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2. In the diagram, $AB = 8$ cm, $AC = 11$ cm and $DC = 6.5$ cm. $B\hat{A}D = 26°$ and $D\hat{A}C = 30°$.



(a) Calculate BC.

Answer ..... cm [4]

(b) Calculate the obtuse angle ADC.

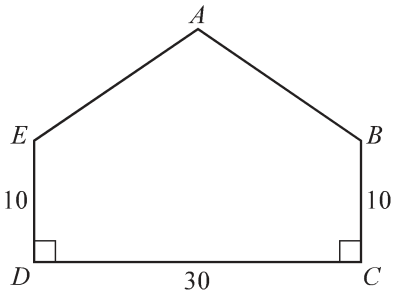
Answer ..... [3]

(c) Find the percentage of triangle ABC that has been shaded.

Answer ..... % [4]

[June/2015/P22/Q5]

3. (a) $ABCDE$ is a pentagon with one line of symmetry.
$BC = DE = 10$ cm, $DC = 30$ cm and $B\hat{C}D = C\hat{D}E = 90°$.
The shortest distance between A and DC is 22 cm.



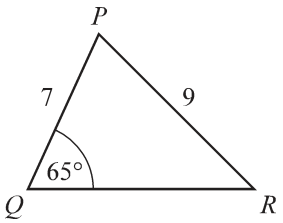
(i) Calculate AB.

Answer ..... cm [2]

(ii) Calculate $A\hat{B}C$.

Answer ..... [3]

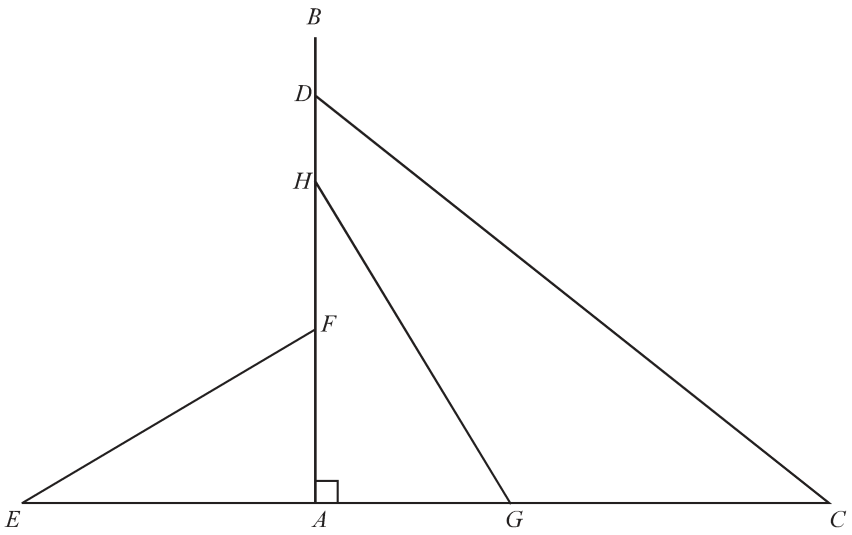
(b) In triangle PQR, $PQ = 7$ cm, $PR = 9$ cm and $P\hat{Q}R = 65°$.
Calculate $P\hat{R}Q$.



Answer ..... [3]

[Nov/2015/P21/Q2]

4.



The diagram shows a vertical radio mast, *AB*.
Three of the wires that hold the mast in place are attached to it at *F*, *H* and *D*.
The base *A* of the mast, and the ends *E*, *G* and *C* of the wires are in a straight line on horizontal ground.

(a) The wire *CD* has length 65 m. It is attached to the mast at *D* where $AD = 40$ m.
Calculate *AC*.

Answer ..... m [2]

(b) The wire *EF* makes an angle of 25° with the horizontal and is of length 30 m.
Calculate *AF* .

Answer ..... m [2]

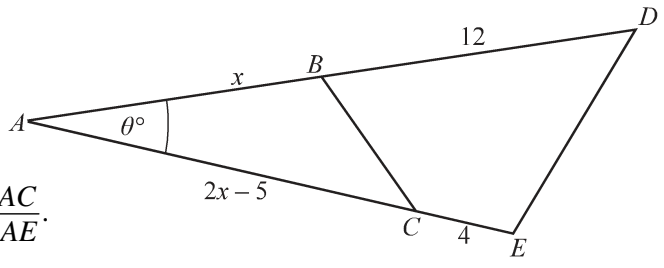
(c) $AH = 35$ m.
The wire *HG* makes an angle of 30º with the mast *AB*.
Calculate *HG*.

Answer ..... m [3]

[Nov/2015/P22/Q5]

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5. *ABD* and *ACE* are straight lines.
BD = 12 cm and *CE* = 4 cm.
AB = *x* cm and *AC* = (2*x* – 5) cm.
Angle $BAC = \theta°$.



(a) Show that $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{AB \times AC}{AD \times AE}$.

[2]

(b) It is given that $\frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{1}{3}$.

Using the result from part **(a)**, form an equation in x and show that it simplifies to $2x^2 - 19x + 6 = 0$.

[3]

(c) (i) Solve the equation $2x^2 - 19x + 6 = 0$, giving your answers correct to 2 decimal places.

Answer x = ... or ..... [3]

(ii) State, with a reason, which of these solutions does **not** apply to triangle *ABC*.

Answer ..... [1]

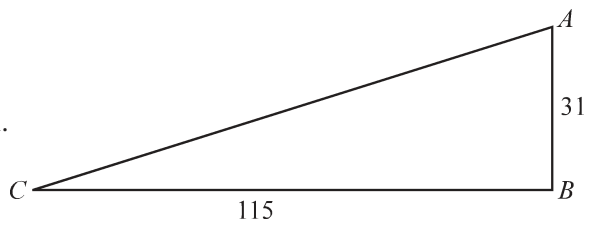
(d) Given that $\theta = 25$, calculate *BC*.

Answer ..... cm [3]

[Nov/2015/P22/Q7]

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6. (a) *AB* is vertical and *CB* is horizontal.
AB = 31 m and *CB* = 115 m.
Calculate the angle of depression of *C* from *A*.



Answer ..... [3]

ANSWERS

Topic 23 - Bearings and Trigonometry

1. **(a)** **(i)** $\dfrac{8\sin 54°}{\sin 18°} = 20.94$

(ii) $\sqrt{4.73^2 - 1.65\sin 43°} = 4.61$

(b) **(i)** Using cosine rule,

$16^2 = x^2 + (2x+3)^2 - 2(x)(2x+3)\cos 60°$

$256 = x^2 + (4x^2 + 12x + 9) - 2x^2 - 3x$

$\Rightarrow\ 3x^2 + 9x - 247 = 0$

(ii) $x = \dfrac{-9 \pm \sqrt{9^2 - 4(3)(-247)}}{2(3)}$

$\Rightarrow\ x = \dfrac{-9 \pm \sqrt{3045}}{6}$

$\therefore\ x = 7.70 \quad \text{or} \quad -10.70$

(iii) $AB = 7.70$ cm

$AC = 2(7.70) + 3 = 18.4$ cm

(iv) Area of $\Delta ABC = \dfrac{1}{2}(7.70)(18.4)\sin 60°$

$= 61.35\ \text{cm}^2$

2. **(a)** In ΔABD, using cosine rule,

$BC = \sqrt{(11)^2 + (8)^2 - 2(11)(8)\cos 56°}$

$= 9.3049 \approx 9.30$ cm

(b) $\dfrac{\sin A\hat{D}C}{11} = \dfrac{\sin 30°}{6.5} \quad \Rightarrow \quad A\hat{D}C = 57.8°$

Obtuse angle $ADC = 180° - 57.8° = 122°$

(c) $A\hat{C}D = 180° - 122° - 30° = 28°$

Area of $\Delta ADC = \dfrac{1}{2}(11)(6.5)\sin 28°$

$= 16.784\ \text{cm}^2$

Area of $\Delta ABC = \dfrac{1}{2}(11)(8)\sin 56°$

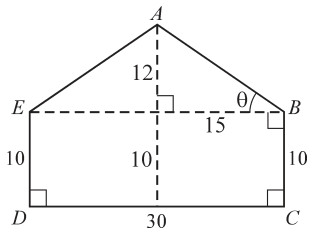
$= 36.478\ \text{cm}^2$

percentage shaded $= \dfrac{16.784}{36.478} \times 100 = 46\%$

3. **(a)** **(i)** AB

$= \sqrt{12^2 + 15^2}$

$= 19.21$ cm



(ii) $\tan\theta° = \dfrac{12}{15}$

$\Rightarrow\ \theta° = 38.7°$

$\therefore\ A\hat{B}C = 38.7° + 90° = 128.7°$

(b) $\dfrac{\sin P\hat{R}Q}{7} = \dfrac{\sin 65°}{9} \quad \Rightarrow \quad P\hat{R}Q = 44.8°$

4. **(a)** $AC = \sqrt{(65)^2 - (40)^2} = 51.235 \approx 51.2$ m.

(b) $\sin 25° = \dfrac{AF}{30} \quad \Rightarrow \quad AF = 12.678 \approx 12.7$ m

(c) $\cos 30° = \dfrac{35}{HG} \quad \Rightarrow \quad HG = 40.4$ m

5. **(a)** $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{\frac{1}{2}\times AB\times AC\times\sin\theta}{\frac{1}{2}\times AD\times AE\times\sin\theta}$

$\Rightarrow\ \dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{AB\times AC}{AD\times AE}$

(b) $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{AB\times AC}{AD\times AE} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{x\times(2x-5)}{(x+12)\times(2x-5+4)} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{2x^2 - 5x}{2x^2 + 23x - 12} = \dfrac{1}{3}$

$\Rightarrow\ 3(2x^2 - 5x) = 2x^2 + 23x - 12$

$\Rightarrow\ 4x^2 - 38x + 12 = 0$

$\Rightarrow\ 2x^2 - 19x + 6 = 0$

(c) **(i)** $x = \dfrac{-(-19) \pm \sqrt{(-19)^2 - 4(2)(6)}}{2(2)}$

$= \dfrac{19 \pm \sqrt{313}}{4}$

$\Rightarrow\ x = 9.17 \quad \text{or} \quad 0.33$ (2dp).

(ii) For $x = 0.33$,
$AC = 2(0.33) - 5 = -4.34$
Length cannot be negative, therefore $x = 0.33$ does not apply to ΔABC.

(d) $AB = 9.17, \quad AC = 13.34$
Using cosine rule, BC
$= \sqrt{(9.17)^2 + (13.34)^2 - 2(9.17)(13.34)\cos 25°}$
$= \sqrt{40.312} = 6.35$ cm

6. **(a)** $\tan A\hat{C}B = \dfrac{31}{115} \Rightarrow A\hat{C}B = 15.1°$
$\therefore$ Angle of depression of C from $A = 15.1°$

(b) **(i)** $\sin L\hat{J}K = \dfrac{354}{1100} \Rightarrow L\hat{J}K = 18.8°$

(ii) Bearing of K from $J = 270° - 18.8°$
$= 251.2°$

7. **(a)** **(i)** $AC = \sqrt{(2)^2 + (5)^2} = 5.385 \approx 5.39$ m.

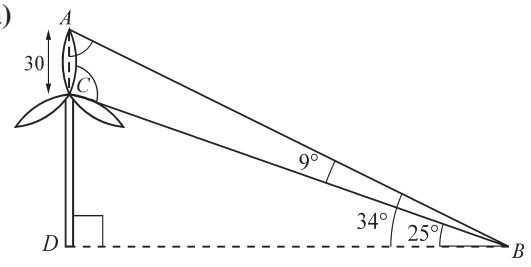
(ii) $\sin 15° = \dfrac{CE}{2} \Rightarrow CE = 0.518$ m.

(iii) By pythagoras, $AE = \sqrt{AC^2 - CE^2}$
$AE = \sqrt{(5.385)^2 - (0.518)^2} = 5.36$ m.
In ΔFAE, $\sin F\hat{A}E = \dfrac{5}{AE}$
$\Rightarrow \sin F\hat{A}E = \dfrac{5}{5.36} \Rightarrow F\hat{A}E = 68.9°$

(b) **(i)** Using cosine rule,
$\cos\theta = \dfrac{9^2 + 6^2 - 10^2}{2(9)(6)} \Rightarrow \theta = 80.9°$

(ii) $c^2 > (a^2 + b^2)$.

8. **(a)**



$A\hat{C}B$ is an exterior angle to ΔBCD
$\therefore A\hat{C}B = 25° + 90° = 115°$
Using sine rule on ΔABC,
$\dfrac{AB}{\sin 115°} = \dfrac{30}{\sin 9°} \Rightarrow AB = 173.8 \approx 174$ m.

(b) Using cosine rule,
$\cos D\hat{F}E = \dfrac{(75)^2 + (180)^2 - (130)^2}{2(75)(180)}$
$\Rightarrow D\hat{F}E = 38.52°$
Angle of depression of E from F
$= 90° - 38.52° = 51.5°$

(c) **(i)** After one revolution, distance covered by P is the circumference of a circle of radius 30 m. So distance travelled $= 2\pi(30) = 188.495 \approx 188$ m.

(ii) Distance travelled in 15 revolutions
$= 188.495 \times 15 = 2827.425$ m.
Speed of $P = \dfrac{2827.425 \times 60}{1000} \approx 170$ km/h.

(iii) Distance travelled by $Q = 2\pi r$
$\Rightarrow 90 = 2\pi r \Rightarrow r = 14.32$
Length of $PQ = 30 - 14.32 \approx 15.7$ m.

9. **(a)** Bearing of D from $A = 180° + 55° + 48°$
$= 283°$

(b) Bearing of A from $C = 055°$

(c) $A\hat{B}C = 180° - 55° - 51° = 74°$
Using sine rule on ΔABC,
$\dfrac{AB}{\sin 51°} = \dfrac{19}{\sin 74°} \Rightarrow AB = 15.36$ km.

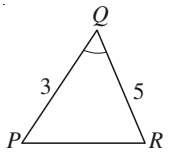
(d) By cosine rule,
$DC = \sqrt{19^2 + 27^2 - 2(19)(27)\cos 48°}$
$= \sqrt{403.472} = 20.1$ km

(e) Draw a line CX from C to DA, such that $CX \perp DA$. Now in ΔACX,
$\cos 48° = \dfrac{AX}{19} \Rightarrow AX = 12.713$ km.
$DX = 27 - 12.713 = 14.287$ km
Time taken from D to $A = 216$ minutes
time taken from D to $X = \dfrac{216}{27} \times 14.287$
$= 114$ minutes.

10. (a) (i) $AD = 3\cos 27° = 2.67$ m.

(ii) $\sin 41° = \dfrac{3}{CD} \Rightarrow CD = 4.57$ m

(b) $6 = \dfrac{1}{2}(3)(5)\sin P\hat{Q}R$



$\Rightarrow \sin P\hat{Q}R = \dfrac{12}{15}$

$\Rightarrow P\hat{Q}R = 53.1°$ or $126.9°$

11. (a) In ΔABC, using cosine rule,

$\cos A\hat{C}B = \dfrac{(70)^2 + (110)^2 - (65)^2}{2(70)(110)}$

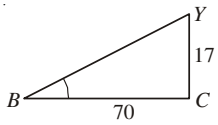
$\Rightarrow A\hat{C}B = 34°$

Bearing of A from $C = 360° - 34° = 326°$

(b) $A\hat{C}D = 180° - 70° - 58° = 52°$

Now, $\dfrac{AD}{\sin 52°} = \dfrac{110}{\sin 70°} \Rightarrow AD = 92.2$ m

(c) (i) Consider ΔBCY,



$\tan C\hat{B}Y = \dfrac{17}{70}$

$\Rightarrow C\hat{B}Y = 13.65°$

$\therefore$ angle of elevation from $B = 13.7°$

(ii) Av. speed $= \dfrac{110}{24}$ m/s

$= \dfrac{110}{24} \times \dfrac{3600}{1000} = 16.5$ km/h

12. (a) (i) $BC = \sqrt{(12)^2 + (8)^2} = 14.42$ km.

(ii) In ΔABD,

$\tan A\hat{D}B = \dfrac{12}{15} \Rightarrow A\hat{D}B = 38.7°$

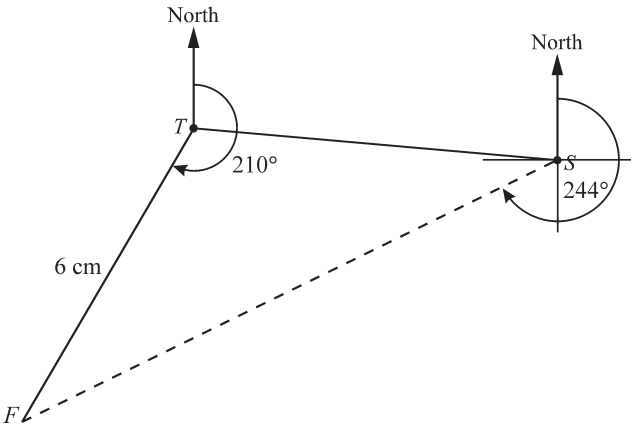
Bearing of A from $D = 90° + 38.7°$

$= 128.7° \approx 129°$

(b) (i) On diagram $TS = 6.4$ cm

Actual distance $= 6.4 \times 75 = 480$ m.

(ii)



(iii) Bearing of F from $S = 244°$

13. (a) Interior angle $= \dfrac{(9-2)180°}{9} = 140°$

(b) (i) $BE = \sqrt{7^2 + 18^2 - 2(7)(18)\cos 115°}$

$= \sqrt{479.4998} = 21.9$ cm

(ii) $\dfrac{\sin D\hat{B}C}{11} = \dfrac{\sin 28°}{16} \Rightarrow D\hat{B}C = 18.8°$

(iii) $18 + 16 + 11 + DE + 7 = 62$

$\Rightarrow DE = 10$ cm

Area of $\Delta DEB = 109$ cm^2

$\Rightarrow \dfrac{1}{2}(21.9)(10)\sin D\hat{E}B = 109$

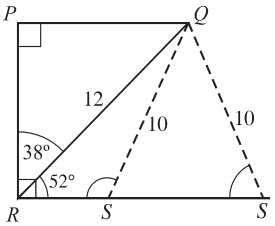
$\Rightarrow D\hat{E}B = 84.5°$

$\therefore$ Obtuse $D\hat{E}B = 180° - 84.5° = 95.5°$

14. (a) $PQ = 12\sin 38° = 7.39$ cm.

(b) $Q\hat{R}S = 90° - 38°$

$= 52°$



$\dfrac{\sin Q\hat{S}R}{12} = \dfrac{\sin 52°}{10}$

$\Rightarrow Q\hat{S}R = 71.0°$

or $180° - 71° = 109°$

15. (a) Draw a line BH from B to ED, such that BH is parallel to CD. Now in ΔEBH,

$HB = DC = \sqrt{(15.1)^2 - (2)^2} = 14.97$ m.

(b) $\cos E\hat{A}B = \dfrac{9^2 + 11^2 - 15.1^2}{2(9)(11)} \Rightarrow E\hat{A}B = 97.5°$