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UNSOLVED EXAM PAPERS

MATHEMATICS

(P1, P3, P4, P5 - All Variants)

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form	**Topic By Topic**
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CONTENTS

PURE MATHEMATICS 1 (PAPER 1)

TOPIC 5

Trigonometry

1. **(i)** Show that the equation $2\tan^2\theta\ \sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta\ \sin^2\theta = 1$ for $0° \le \theta \le 360°$. [4]

[J11/P11/Q5]

2. **(i)** Prove the identity $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv 1 + \dfrac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} = 4$, for $0° \le \theta \le 360°$. [3]

[J11/P12/Q5]

3. **(i)** Prove the identity $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 \equiv \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 = \dfrac{2}{5}$, for $0° \le \theta \le 360°$. [4]

[J11/P13/Q8]

4. **(i)** Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \le \theta \le 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \le \theta \le 2\pi$. [1]

(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \le \theta \le 20\pi$. [1]

[N11/P11/Q3]

5. **(i)** Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0° \le x \le 180°$. [3]

(ii) Verify that $x = 30°$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0° \le x \le 180°$. [2]

(iii) Hence state the set of values of x, for $0° \le x \le 180°$, for which $\sin x < \cos 2x$. [2]

[N11/P12/Q5]

6. **(i)** Given that, $3\sin^2 x - 8\cos x - 7 = 0$. Show that, for real values of x, $\cos x = -\frac{2}{3}$. [3]

(ii) Hence solve the equation $3\sin^2(\theta + 70°) - 8\cos(\theta + 70°) - 7 = 0$ for $0° \le \theta \le 180°$. [4]

[N11/P13/Q5]

7. Solve the equation $\sin 2x = 2\cos 2x$, for $0° \le x \le 180°$. [4]

[J12/P11/Q1]

8. **(i)** Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [2]

(ii) Solve the equation $\frac{2}{\sin x \cos x} = 1 + 3\tan x$, for $0° \le x \le 180°$. [4]

[J12/P12/Q5]

9. **(i)** Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta\sin^2\theta$. [3]

(ii) Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [1]

[J12/P13/Q1]

10. **(i)** Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \le x \le 360°$. [5]

(ii) How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \le x \le 1080°$. [1]

[J12/P13/Q4]

11. **(i)** Solve the equation $2\cos^2\theta = 3\sin\theta$, for $0° \le \theta \le 360°$. [4]

(ii) The smallest positive solution of the equation $2\cos^2(n\theta) = 3\sin(n\theta)$, where n is a positive integer, is 10°. State the value of n and hence find the largest solution of this equation in the interval $0° \le \theta \le 360°$. [3]

[N12/P11/Q7]

12. **(i)** Show that the equation $2\cos x = 3\tan x$ can be written as a quadratic equation in $\sin x$. [3]

(ii) Solve the equation $2\cos 2y = 3\tan 2y$, for $0° \le y \le 180°$. [4]

[N12/P12/Q6]

13. Solve the equation $7\cos x + 5 = 2\sin^2 x$, for $0° \le x \le 360°$. [4]

[N12/P13/Q3]

14. **(i)** Show that $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} \equiv \frac{1}{\sin^2\theta - \cos^2\theta}$. [3]

(ii) Hence solve the equation $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} = 3$, for $0° \le \theta \le 360°$. [4]

[J13/P11/Q5]

15. It is given that $a = \sin\theta - 3\cos\theta$ and $b = 3\sin\theta + \cos\theta$, where $0° \le \theta \le 360°$.

(i) Show that $a^2 + b^2$ has a constant value for all values of θ. [3]

(ii) Find the values of θ for which $2a = b$. [4]

[J13/P12/Q5]

16. **(i)** Express the equation $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. [2]

(ii) Solve the equation $2\cos^2\theta = \tan^2\theta$ for $0 \le \theta \le \pi$, giving solutions in terms of π. [3]

[J13/P13/Q3]

17. **(i)** Sketch, on the same diagram, the curves $y = \sin 2x$ and $y = \cos x - 1$ for $0 \le x \le 2\pi$. [4]

ANSWERS

Topic 5 - Trigonometry

1. **(i)** $2\tan^2\theta\ \sin^2\theta = 1$

$\Rightarrow \dfrac{2\sin^2\theta}{\cos^2\theta}\times\sin^2\theta = 1$

$\Rightarrow 2\sin^4\theta = \cos^2\theta$

$\Rightarrow 2\sin^4\theta + \sin^2\theta - 1 = 0$

(ii) $2\sin^4\theta + \sin^2\theta - 1 = 0$

$\Rightarrow (2\sin^2\theta - 1)(\sin^2\theta + 1) = 0$

$\Rightarrow \sin\theta = \pm\dfrac{1}{\sqrt{2}}$ or $in^2\theta = -1$ (impossible)

$\Rightarrow \theta = 45°,\ 135°,\ 225°,\ 315°$

2. **(i)** $\text{L.H.S} = \dfrac{\cos\theta}{\tan\theta(1-\sin\theta)}$

$= \dfrac{\cos\theta}{\frac{\sin\theta}{\cos\theta}(1-\sin\theta)}$

$= \dfrac{\cos^2\theta}{\sin\theta(1-\sin\theta)}$

$= \dfrac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$

$= \dfrac{1+\sin\theta}{\sin\theta} = 1 + \dfrac{1}{\sin\theta}$

(ii) $1 + \dfrac{1}{\sin\theta} = 4 \Rightarrow \sin\theta = \dfrac{1}{3}$

$\therefore\ \theta = 19.5°,\ 160.5°$

3. **(i)** $\text{L.H.S.} \equiv \left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2$

$= \left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

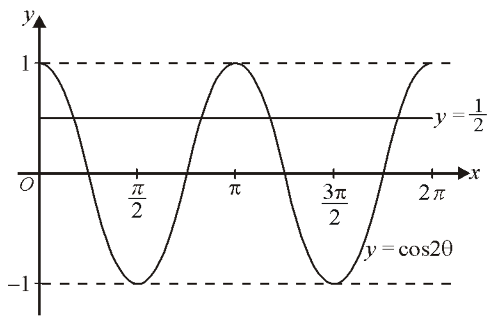
$= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta} = \dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\dfrac{1-\cos\theta}{1+\cos\theta} = \dfrac{2}{5}$

$\Rightarrow 7\cos\theta = 3 \Rightarrow \cos\theta = \dfrac{3}{7}$

$\Rightarrow \theta = 64.6°,\ 295.4°$

4. **(i)**

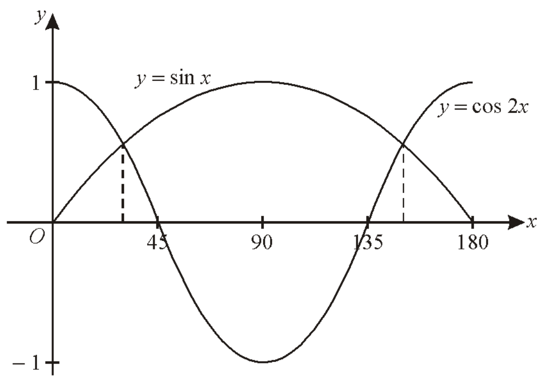


(ii) $2\cos 2\theta - 1 = 0 \Rightarrow \cos 2\theta = \dfrac{1}{2}$

$\therefore$ Number of real roots = 4

(iii) Number of roots = 20

5. **(i)**



(ii) $\sin 30° = \dfrac{1}{2}$, and $\cos 2(30°) = \dfrac{1}{2}$

From graph, $x = 150°$ is the other root,

(iii) From graph, for $\sin x < \cos 2x$,

$0° \le x < 30°$, and $150° < x \le 180°$.

6. **(i)** $3\sin^2 x - 8\cos x - 7 = 0$

$\Rightarrow 3(1-\cos^2 x) - 8\cos x - 7 = 0$

$\Rightarrow 3\cos^2 x + 8\cos x + 4 = 0$

$\Rightarrow (3\cos x + 2)(\cos x + 2) = 0$

$\therefore\ \cos x = -\dfrac{2}{3}$

(ii) $3\sin^2(\theta + 70°) - 8\cos(\theta + 70°) - 7 = 0$

$\Rightarrow \cos(x + 70°) = -\dfrac{2}{3}$

$\Rightarrow x + 70° = 131.8°,\ 228.2°$

$\Rightarrow x = 61.8°,\ 158.2°$

7. $\sin 2x = 2\cos 2x \quad \Rightarrow \quad \tan 2x = 2$

$\Rightarrow 2x = 63.4, \; 243.4 \Rightarrow x = 31.7°, \; 121.7°$

8. (i) $\text{L.H.S} = \tan x + \dfrac{1}{\tan x}$

$= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \dfrac{1}{\sin x \cos x}$

(ii) $\dfrac{2}{\sin x \cos x} = 1 + 3\tan x,$

$\Rightarrow 2\left(\tan x + \dfrac{1}{\tan x}\right) = 1 + 3\tan x$

$\Rightarrow \tan^2 x + \tan x - 2 = 0$

$\Rightarrow (\tan x + 2)(\tan x - 1) = 0$

$\Rightarrow \tan x = -2 \quad \text{or} \quad \tan x = 1$

$\therefore \; x = 45°, \; 116.6°$

9. (i) $\text{L.H.S.} \equiv \tan^2\theta - \sin^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \sin^2\theta\left(\dfrac{1}{\cos^2\theta} - 1\right)$

$= \sin^2\theta\left(\dfrac{1-\cos^2\theta}{\cos^2\theta}\right) = \tan^2\theta\sin^2\theta$

(ii) Since, $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

$\Rightarrow \tan^2\theta - \sin^2\theta > 0$

$\Rightarrow \tan^2\theta > \sin^2\theta$

$\Rightarrow \tan\theta > \sin\theta \;$ for $0° < \theta < 90°$.

10. (i) $\sin 2x + 3\cos 2x = 0 \quad \Rightarrow \quad \tan 2x = -3$

$\Rightarrow 2x = 108.4°, \; 288.43, \; 468.4, \; 648.4$

$\Rightarrow x = 54.2°, \; 144.2°, \; 234.2, \; 324.2$

(ii) Number of solutions = 12

11. (i) $2\cos^2\theta = 3\sin\theta$

$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 2)$

$\Rightarrow \theta = 30° \;$ or $\; 150°$

(ii) Smallest positive value of $\theta = 10°$

$\Rightarrow n\theta = 30° \quad \Rightarrow \quad n = \dfrac{30}{10} = 3$

For largest solution of θ,

$3\theta = 720° + 150° \quad \Rightarrow \quad \theta = \dfrac{870°}{3} = 290°$

12. (i) $2\cos x = 3\tan x$

$\Rightarrow 2\cos x = 3\left(\dfrac{\sin x}{\cos x}\right)$

$\Rightarrow 2\cos^2 x = 3\sin x$

$\Rightarrow 2(1 - \sin^2 x) = 3\sin x$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

(ii) $2\cos 2y = 3\tan 2y$

$\Rightarrow 2\sin^2 2y + 3\sin 2y - 2 = 0$

$\Rightarrow (2\sin 2y - 1)(\sin 2y + 2) = 0$

$\Rightarrow y = 15°, \; 75°$

13. $7\cos x + 5 = 2\sin^2 x$

$\Rightarrow 2\cos^2 x + 7\cos x + 3 = 0$

$\Rightarrow (2\cos x + 1)(\cos x + 3) = 0$

$\Rightarrow x = 120°, \; 240°$

14. (i) $\text{L.H.S.} \equiv \dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta}$

$= \dfrac{\sin\theta(\sin\theta - \cos\theta) + \cos\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$

$= \dfrac{1}{\sin^2\theta - \cos^2\theta}$

(ii) $\dfrac{1}{\sin^2\theta - \cos^2\theta} = 3$

$\Rightarrow \dfrac{1}{1 - 2\cos^2\theta} = 3$

$\Rightarrow \cos^2\theta = \dfrac{1}{3} \quad \Rightarrow \quad \cos\theta = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = 54.7, \; 125.3°, \; 234.7°, \; 305.3°$

15. (i) $a^2 + b^2$

$= (\sin\theta - 3\cos\theta)^2 + (3\sin\theta + \cos\theta)^2$

$= \sin^2\theta - 6\sin\theta\cos\theta + 9\cos^2\theta + 9\sin^2\theta + 6\sin\theta\cos\theta + \cos^2\theta$

$= 10(\sin^2\theta + \cos^2\theta) = 10$

(ii) $2a = b$

$\Rightarrow 2(\sin\theta - 3\cos\theta) = 3\sin\theta + \cos\theta$

$\Rightarrow -\sin\theta = 7\cos\theta \quad \Rightarrow \quad \tan\theta = -7$

$\therefore \; \theta = 98.1°, \; 278.1°$

16. (i) $2\cos^2\theta = \tan^2\theta$

$\Rightarrow 2\cos^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow 2\cos^4\theta + \cos^2\theta - 1 = 0$

PURE MATHEMATICS 3 (PAPER 3)

TOPIC 19

Differential Equations

1. The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation and hence find the value of y when $x = 0.5$, giving your answer correct to 2 decimal places. [8]

[J12/P31/Q7]

2. The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{2x+y},$$

and $y = 0$ when $x = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

[J12/P32/Q5]

3. In a certain chemical process a substance A reacts with another substance B. The masses in grams of A and B present at time t seconds after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.6xy$ and $x = 5e^{-3t}$. When $t = 0$, $y = 70$.

(i) Form a differential equation in y and t. Solve this differential equation and obtain an expression for y in terms of t. [6]

(ii) The percentage of the initial mass of B remaining at time t is denoted by p. Find the exact value approached by p as t becomes large. [2]

[J12/P33/Q5]

4. The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

[N12/P31&32/Q6]

5. The variables x and y are related by the differential equation, $(x^2 + 4)\frac{dy}{dx} = 6xy.$

It is given that $y = 32$ when $x = 0$. Find an expression for y in terms of x. [6]

[N12/P33/Q4]

6. Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is V cm^3. The liquid is flowing into the tank at a constant rate of 80 cm^3 per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to kV cm^3 per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80e^{-kt}).$$ [7]

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation $k = \frac{4 - 4e^{-15k}}{25}$.
Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

[J13/P31/Q10]

7. **(i)** Express $\frac{1}{x^2(2x+1)}$ in the form $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x+1}$. [4]

(ii) The variables x and y satisfy the differential equation $y = x^2(2x+1)\frac{dy}{dx}$, and $y = 1$ when $x = 1$.
Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

[J13/P32/Q8]

8. The variables x and t satisfy the differential equation $t\frac{dx}{dt} = \frac{k - x^3}{2x^2}$,

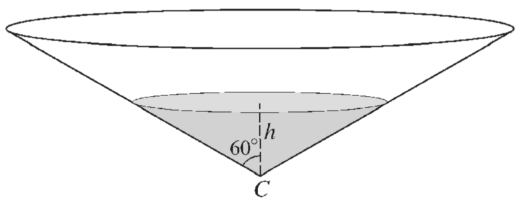
for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x as t becomes large. [1]

[J13/P33/Q8]

9.



A tank containing water is in the form of a cone with vertex C. The axis is vertical and the semivertical angle is 60°, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is H. At this instant, a tap at C is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where h is the depth of water at time t. The tank becomes empty when $t = 60$.

(i) Show that h and t satisfy a differential equation of the form $\frac{dh}{dt} = -Ah^{-\frac{3}{2}}$, where A is a positive constant. [4]

(ii) Solve the differential equation given in part **(i)** and obtain an expression for t in terms of h and H. [6]

ANSWERS

Topic 19 - Differential Equations

1. $\dfrac{dy}{dx}=\dfrac{6xe^{3x}}{y^2} \quad\Rightarrow\quad y^2\,dy=6\,x\,e^{3x}\,dx$

Integrate both sides to obtain,

$$\frac{y^3}{3}=6\left(\frac{1}{3}xe^{3x}-\frac{1}{3}\int e^{3x}\,dx\right)$$

$$\Rightarrow\ \frac{y^3}{3}=2xe^{3x}-\frac{2}{3}e^{3x}+C$$

Subst. $x=0,\quad y=2,$ to get, $C=\dfrac{10}{3}$

$\therefore\ y=\left(6xe^{3x}-2e^{3x}+10\right)^{\frac{1}{3}}$, At $x=0.5,\ \ y=2.44$

2. $\dfrac{dy}{dx}=e^{2x+y} \quad\Rightarrow\quad e^{-y}\,dy=e^{2x}\,dx$

Integrate both sides to get, $-e^{-y}=\dfrac{1}{2}e^{2x}+K$

when $x=0,\quad y=0\quad\Rightarrow\quad K=-\dfrac{3}{2}$

$\therefore\quad -e^{-y}=\dfrac{1}{2}e^{2x}-\dfrac{3}{2}\quad\Rightarrow\quad 2e^{-y}=3-e^{2x}$

Take ln on both sides, and solve to obtain,

$$y=\ln\left(\frac{2}{3-e^{2x}}\right)$$

3. **(i)** $\dfrac{dy}{dt}=-0.6\,x\,y\quad\Rightarrow\quad\dfrac{dy}{dt}=-0.6(5e^{-3t})\,y$

$\Rightarrow\ \dfrac{1}{y}dy=-3e^{-3t}\,dt$

Integrate both sides to get, $\ln y=e^{-3t}+C$

When $t=0,\quad y=70\quad\Rightarrow\quad C=\ln 70-1$

$\therefore\quad \ln y=e^{-3t}+\ln 70-1$

solve it to get, $y=70e^{(e^{-3t}-1)}$

(ii) $p=\dfrac{70e^{(e^{-3t}-1)}}{70}\times 100\quad\Rightarrow\quad p=e^{(e^{-3t}-1)}\times 100$

As $t\to\infty,\quad e^{-3t}\to 0$

$\therefore\quad p=e^{-1}\times 100=\dfrac{100}{e}$

4. $x\dfrac{dy}{dx}=1-y^2\quad\Rightarrow\quad\dfrac{1}{(1+y)(1-y)}\,dy=\dfrac{1}{x}\,dx$

Resolving into partial fractions,

$$\frac{1}{(1+y)(1-y)}\equiv\frac{A}{1+y}+\frac{B}{1-y}$$

$\Rightarrow\ A=\dfrac{1}{2},\quad B=\dfrac{1}{2}$

$\therefore\quad\left(\dfrac{1}{2(1+y)}+\dfrac{1}{2(1-y)}\right)dy=\dfrac{1}{x}\,dx$

Integrate to get, $\dfrac{1}{2}\ln\left(\dfrac{1+y}{1-y}\right)=\ln x+K$

When $x=2,\quad y=0\quad\Rightarrow\quad K=-\ln 2$

$\therefore\quad\dfrac{1}{2}\ln\left(\dfrac{1+y}{1-y}\right)=\ln x-\ln 2$

solve it to get, $y=\dfrac{x^2-4}{x^2+4}$

5. $(x^2+4)\dfrac{dy}{dx}=6xy\quad\Rightarrow\quad\dfrac{1}{y}dy=\dfrac{6x}{x^2+4}\,dx$

Integrate to get, $\ln y=3\ln(x^2+4)+C$

When, $y=32,\quad x=0\quad\Rightarrow\quad C=-\ln 2$

$\therefore\quad\ln y=3\ln(x^2+4)-\ln 2$

$\Rightarrow\ y=\dfrac{(x^2+4)^3}{2}$

6. **(i)** $\dfrac{dV}{dt}=80-kV\quad\Rightarrow\quad\dfrac{1}{80-kV}\,dV=dt$

Integrate to get, $\ln|80-kV|=-kt+C$

At, $t=0,\ V=0\quad\Rightarrow\quad C=\ln 80$

$\therefore\quad\ln|80-kV|=-kt+\ln 80$

$\Rightarrow\ \dfrac{80-kV}{80}=e^{-kt}\quad\Rightarrow\quad V=\dfrac{1}{k}(80-80e^{-kt})$

(ii) Subst. $V=500,\ t=15$ into (i), to obtain,

$$k=\frac{4-4e^{-15k}}{25}$$

Iterative formula: $k_{n+1}=\dfrac{4-4e^{-15k_n}}{25}$

Initial value, $k_1=0.1$

$\Rightarrow\ k_2=\dfrac{4-4e^{-15(0.1)}}{25}=0.1243$

$k_3=0.1352,\ \ k_4=0.1389,\ \ k_5=0.1401,$

$k_6=0.1404,\quad\therefore k=0.14$

(iii) $V=\dfrac{1}{0.14}(80-80e^{-(0.14)t})$

At, $t=20,\quad V=537\text{ cm}^3$

Now, as $t\to\infty,\quad e^{-0.14t}\to 0$

$\therefore\quad V=\dfrac{1}{0.14}(80-0)=571\text{ cm}^3$

7. **(i)** $\dfrac{1}{x^2(2x+1)} \equiv \dfrac{A}{x^2}+\dfrac{B}{x}+\dfrac{C}{2x+1}$

$\Rightarrow A=1,\quad B=-2,\quad C=4$

$\therefore\quad \dfrac{1}{x^2(2x+1)} \equiv \dfrac{1}{x^2}-\dfrac{2}{x}+\dfrac{4}{2x+1}$

(ii) $y=x^2(2x+1)\dfrac{dy}{dx} \quad\Rightarrow\quad \dfrac{1}{y}\,dy=\dfrac{1}{x^2(2x+1)}\,dx$

$\Rightarrow \dfrac{1}{y}\,dy=\left(\dfrac{1}{x^2}-\dfrac{2}{x}+\dfrac{4}{2x+1}\right)dx$

integrate both sides to obtain,

$\ln y=-\dfrac{1}{x}-2\ln x+2\ln|2x+1|+K$

when $x=1,\ y=1 \quad\Rightarrow\quad K=1-2\ln 3$

$\therefore\quad \ln y=-\dfrac{1}{x}-2\ln x+2\ln|2x+1|+1-2\ln 3$

$\Rightarrow \ln y=-\dfrac{1}{x}+1+2\ln\left|\dfrac{2x+1}{3x}\right|$

Subst. $x=2$, and solve to get, $y=\dfrac{25}{36}e^{\frac{1}{2}}$

8. **(i)** $t\,\dfrac{dx}{dt}=\dfrac{k-x^3}{2x^2} \quad\Rightarrow\quad \dfrac{2x^2}{k-x^3}\,dx=\dfrac{1}{t}\,dt$

Integrate to get, $-\dfrac{2}{3}\ln(k-x^3)=\ln t+C$

When $t=1,\quad x=1 \quad\Rightarrow\quad C=-\dfrac{2}{3}\ln(k-1)$

$\therefore\quad -\dfrac{2}{3}\ln(k-x^3)=\ln t-\dfrac{2}{3}\ln(k-1)$

$\Rightarrow \ln t=\dfrac{2}{3}\ln\dfrac{k-1}{k-x^3}$

Subst. $t=4,\ x=2$, and solve to get, $k=9$

Now, $\ln t=\dfrac{2}{3}\ln\dfrac{8}{9-x^3}$

simplify it to obtain, $x=\left(9-8t^{-\frac{3}{2}}\right)^{\frac{1}{3}}$

(ii) As $t\to\infty,\quad t^{-\frac{3}{2}}\to 0,\quad \therefore\ x=(9)^{\frac{1}{3}}=2.08$

9. **(i)** Let r be the radius of the surface of water

$\therefore\quad \tan 60^\circ=\dfrac{r}{h} \quad\Rightarrow\quad r=\sqrt{3}\,h$

Vol. of water, $V=\dfrac{1}{3}\pi r^2h \quad\Rightarrow\quad V=\pi h^3$

$\therefore\quad \dfrac{dV}{dh}=3\pi h^2$. Given that, $\dfrac{dV}{dt}=-k\sqrt{h}$

applying chain rule, $\dfrac{dV}{dt}=\dfrac{dV}{dh}\times\dfrac{dh}{dt}$

$\Rightarrow \dfrac{dh}{dt}=\dfrac{-k\sqrt{h}}{3\pi h^2}=-Ah^{-\frac{3}{2}}$ (let $A=\dfrac{k}{3\pi}$).

(ii) $\dfrac{dh}{dt}=-Ah^{-\frac{3}{2}} \quad\Rightarrow\quad h^{\frac{3}{2}}\,dh=-A\,dt$

Integrate both sides to get, $\dfrac{2}{5}h^{\frac{5}{2}}=-At+C$

Given that, at $t=0,\ h=H \quad\Rightarrow\quad C=\dfrac{2}{5}H^{\frac{5}{2}}$

$\therefore\quad \dfrac{2}{5}h^{\frac{5}{2}}=-At+\dfrac{2}{5}H^{\frac{5}{2}}$

Also, at $t=60,\ h=0 \quad\Rightarrow\quad A=\dfrac{1}{150}H^{\frac{5}{2}}$

$\therefore$ eq. becomes, $\dfrac{2}{5}h^{\frac{5}{2}}=-\dfrac{1}{150}H^{\frac{5}{2}}t+\dfrac{2}{5}H^{\frac{5}{2}}$

solve the eq. to obtain, $t=60-60\left(\dfrac{h}{H}\right)^{\frac{5}{2}}$

(iii) When, $h=\dfrac{1}{2}H,\quad t=60-60\left(\dfrac{\frac{1}{2}H}{H}\right)^{\frac{5}{2}}=49.4$

10. Re-write the eq. as, $\dfrac{3y^2}{y^3+1}\,dy=4\cos^2 x\,dx$

Integrate to get, $\ln(1+y^3)=2\left(x+\dfrac{1}{2}\sin 2x\right)+C$

When $y=2,\ x=0 \quad\Rightarrow\quad C=\ln 9$

$\therefore\quad \ln(1+y^3)=2x+\sin 2x+\ln 9$

solve it to get, $y=\left(9e^{(2x+\sin 2x)}-1\right)^{\frac{1}{3}}$

Now, for stationary points,

$\dfrac{dy}{dx}=\dfrac{4(y^3+1)\cos^2 x}{3y^2}=0 \quad\Rightarrow\quad 4(y^3+1)\cos^2 x=0$

$\Rightarrow y=-1$ (rej.), or $x=\dfrac{\pi}{2},\ \dfrac{3\pi}{2}$

when $x=\dfrac{\pi}{2},\quad y=\left(9e^{(\pi+0)}-1\right)^{\frac{1}{3}}=5.92$

when $x=\dfrac{3\pi}{2},\quad y=\left(9e^{(3\pi+0)}-1\right)^{\frac{1}{3}}=48.13$

$\therefore$ y-coordinates of A and B are 5.9 and 48.1

11. $\dfrac{dy}{dx}=\dfrac{6ye^{3x}}{2+e^{3x}} \quad\Rightarrow\quad \dfrac{1}{y}\,dy=\dfrac{6e^{3x}}{2+e^{3x}}\,dx$

Integrate to get, $\ln y=2\ln(2+e^{3x})+C$

Given, $y=36$, when $x=0 \quad\Rightarrow\quad C=\ln 4$

$\therefore\quad \ln y=2\ln(2+e^{3x})+\ln 4$

simplify to obtain, $y=4(2+e^{3x})^2$

MECHANICS (PAPER 4)

TOPIC 24

Newton's Laws of Motion

1. An object is released from rest at a height of 125 m above horizontal ground and falls freely under gravity, hitting a moving target P. The target P is moving on the ground in a straight line, with constant acceleration 0.8 m s^{-2}. At the instant the object is released P passes through a point O with speed 5 m s^{-1}. Find the distance from O to the point where P is hit by the object. [4]

[N12/P41/Q1]

2. A, B and C are three points on a line of greatest slope of a plane which is inclined at $\theta°$ to the horizontal, with A higher than B and B higher than C. Between A and B the plane is smooth, and between B and C the plane is rough. A particle P is released from rest on the plane at A and slides down the line ABC. At time 0.8 s after leaving A, the particle passes through B with speed 4 ms^{-1}.

(i) Find the value of θ. [3]

At time 4.8 s after leaving A, the particle comes to rest at C.

(ii) Find the coefficient of friction between P and the rough part of the plane. [5]

[N12/P42/Q5]

3. A particle P is projected vertically upwards, from a point O, with a velocity of 8 m s^{-1}. The point A is the highest point reached by P. Find

(i) the speed of P when it is at the mid-point of OA, [4]

(ii) the time taken for P to reach the mid-point of OA while moving upwards. [2]

[N12/P43/Q3]

4. The top of a cliff is 40 metres above the level of the sea. A man in a boat, close to the bottom of the cliff, is in difficulty and fires a distress signal vertically upwards from sea level. Find

(i) the speed of projection of the signal given that it reaches a height of 5 m above the top of the cliff, [2]

(ii) the length of time for which the signal is above the level of the top of the cliff. [2]

The man fires another distress signal vertically upwards from sea level. This signal is above the level of the top of the cliff for $\sqrt{(17)}$ s.

(iii) Find the speed of projection of the second signal. [3]

[J13/P41/Q3]

5. A string is attached to a block of weight 30 N, which is in contact with a rough horizontal plane. When the string is horizontal and the tension in it is 24 N, the block is in limiting equilibrium.

(i) Find the coefficient of friction between the block and the plane. [2]

The block is now in motion and the string is at an angle of 30° upwards from the plane. The tension in the string is 25 N.

(ii) Find the acceleration of the block. [4]

[J13/P42/Q1]

6. A particle P is released from rest at the top of a smooth plane which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{16}{65}$. The distance travelled by P from the top to the bottom is S metres, and the speed of P at the bottom is 8 ms^{-1}.

(i) Find the value of S and hence find the speed of P when it has travelled $\frac{1}{2}S$ metres. [5]

The time taken by P to travel from the top to the bottom of the plane is T seconds.

(ii) Find the distance travelled by P at the instant when it has been moving for $\frac{1}{2}T$ seconds. [2]

[J13/P42/Q4]

7. A straight ice track of length 50 m is inclined at 14° to the horizontal. A man starts at the top of the track, on a sledge, with speed 8 m s^{-1}. He travels on the sledge to the bottom of the track. The coefficient of friction between the sledge and the track is 0.02. Find the speed of the sledge and the man when they reach the bottom of the track. [4]

[J13/P43/Q1]

8. A particle P is projected vertically upwards from a point on the ground with speed 17 m s^{-1}. Another particle Q is projected vertically upwards from the same point with speed 7 m s^{-1}. Particle Q is projected T seconds later than particle P.

(i) Given that the particles reach the ground at the same instant, find the value of T. [2]

(ii) At a certain instant when both P and Q are in motion, P is 5m higher than Q. Find the magnitude and direction of the velocity of each of the particles at this instant. [6]

[J13/P43/Q5]

9. A cyclist exerts a constant driving force of magnitude F N while moving up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{36}{325}$. A constant resistance to motion of 32 N acts on the cyclist. The total weight of the cyclist and his bicycle is 780 N. The cyclist's acceleration is –0.2 m s^{-2}.

(i) Find the value of F. [4]

The cyclist's speed is 7 m s^{-1} at the bottom of the hill.

(ii) Find how far up the hill the cyclist travels before coming to rest. [2]

[N13/P41/Q3]

10. Particles P and Q are moving in a straight line on a rough horizontal plane. The frictional forces are the only horizontal forces acting on the particles.

(i) Find the deceleration of each of the particles given that the coefficient of friction between P and the plane is 0.2, and between Q and the plane is 0.25. [2]

At a certain instant, P passes through the point A and Q passes through the point B. The distance AB is 5 m. The velocities of P and Q at A and B are 8 m s^{-1} and 3 m s^{-1}, respectively, both in the direction AB.

(ii) Find the speeds of P and Q immediately before they collide. [5]

[N13/P41/Q4]

ANSWERS

Topic 24 - Newton's Laws of Motion

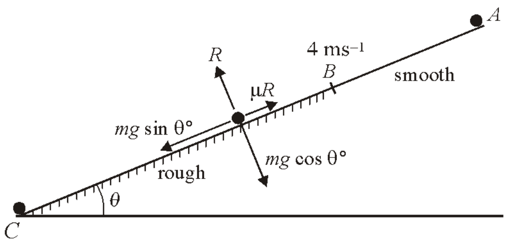
1. Using, $s = ut + \frac{1}{2}gt^2$, time taken for the object to fall from 125 m is,

$125 = 0 + \frac{1}{2}(10)t^2 \quad \Rightarrow \quad t = 5$ s.

$\therefore$ Required distance from O of P is,

$s = 5(5) + \frac{1}{2}(0.8)(5)^2 = 35$ m.

2.



(i) From A to B, using $v = u + at$

$4 = 0 + (g\sin\theta)(0.8) \quad \Rightarrow \quad \theta = 30°$

(ii) From B to C, using $v = u + at$,

$0 = 4 + a(4.8 - 0.8) \quad \Rightarrow \quad a = -1 \text{ ms}^{-2}$

By Newton's 2nd law, $mg\sin\theta - \mu R = ma$

$\Rightarrow \; mg\sin\theta - \mu(mg\cos\theta) = ma$

$\Rightarrow \; g\sin\theta - \mu g\cos\theta = a$

Subst. values of g, θ & a to obtain, $\mu = 0.693$

3. **(i)** Using, $2gs = v^2 - u^2$, height OA is,

$2(-10)s = 0^2 - 8^2 \quad \Rightarrow \quad s = 3.2$ m

Height at mid-point of $OA = \frac{3.2}{2} = 1.6$ m

$\therefore$ Speed at mid-point of OA is,

$2(-10)(1.6) = v^2 - 8^2 \quad \Rightarrow \quad v = 5.66 \text{ ms}^{-1}$

(ii) Using, $v = u + gt$,

$5.66 = 8 - 10t \quad \Rightarrow \quad t = 0.234$ s.

4. **(i)** Using, $2gs = v^2 - u^2$,

$2(-10)(45) = 0^2 - u^2 \quad \Rightarrow \quad u = 30 \text{ ms}^{-1}$

(ii) Using, $s = ut + \frac{1}{2}gt^2$,

$40 = 30t - 5t^2 \quad \Rightarrow \quad t^2 - 6t + 8 = 0$

$\Rightarrow \; t = 2$ s or $t = 4$ s

$\therefore$ time for which the signal is above the top of the cliff $= 4 - 2 = 2$ s.

(iii) The signal moves vertically upwards, reaches its max. height and then starts to fall down. Considering the downward journey of the signal, the height above the top of the cliff can be found as, $s = \frac{1}{2}(10)\left(\frac{\sqrt{17}}{2}\right)^2 = 21.25$

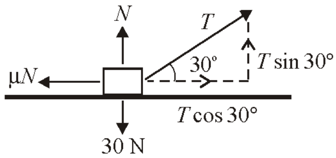
Total height reached $= 40 + 21.25 = 61.25$ m

Using, $2gs = v^2 - u^2$, speed of projection is,

$2(-10)(61.25) = 0^2 - u^2 \quad \Rightarrow \quad u = 35 \text{ ms}^{-1}$

5. **(i)** $T = \mu N \quad \Rightarrow \quad 24 = \mu(30) \quad \Rightarrow \quad \mu = 0.8$

(ii)



$\uparrow: \; N + 25\sin 30° = 30 \quad \Rightarrow \quad N = 17.5$ N

Applying Newton's 2nd law,

$T\cos 30° - \mu N = ma$

$25\cos 30° - (0.8)(17.5) = 3a \quad \Rightarrow \quad a = 2.55 \text{ m/s}^2$

6. **(i)** Acceleration, $a = g\sin\alpha \quad \Rightarrow \quad a = 2.46 \text{ ms}^{-2}$

Using $v^2 - u^2 = 2as$

$(8)^2 - (0)^2 = 2(2.46)(S) \quad \Rightarrow \quad S = 13$ m

$\frac{1}{2}S$ metres $= 6.5$ m

$\therefore \; v^2 - 0 = 2(2.46)(6.5) \quad \Rightarrow \quad v = 5.66 \text{ ms}^{-1}$

(ii) For journey from top to bottom,

using, $v = u + at$, $T = 3.252$ seconds.

$\frac{1}{2}T$ seconds $= \frac{1}{2}(3.252) = 1.626$ seconds.

Apply, $s = ut + \frac{1}{2}at^2$, to obtain, $s = 3.25$ m.

7. By 2nd law, $mg\sin\theta - \mu(mg\cos\theta) = ma$

$\Rightarrow \; g\sin\theta - \mu(g\cos\theta) = a \quad \Rightarrow \quad a = 2.225 \text{ m/s}^2$

Using, $v^2 - u^2 = 2as$

$v^2 - 8^2 = 2(2.225)(50) \quad \Rightarrow \quad v = 16.9 \text{ ms}^{-1}$

8. **(i)** When particles return to the ground, $s = 0$

Using, $s = ut + \frac{1}{2}gt^2$,

For P, $0 = 17t - 5t^2 \Rightarrow t = \frac{17}{5}$ s

For Q, $0 = 7t - 5t^2 \Rightarrow t = \frac{7}{5}$ s

$\therefore T = \frac{17}{5} - \frac{7}{5} = 2$ s

(ii) For P, $h_P = 17(t+2) - 5(t+2)^2$

For Q, $h_Q = 7t - 5t^2$

Given that, $h_P - h_Q = 5$

$\Rightarrow 17(t+2) - 5(t+2)^2 - (7t - 5t^2) = 5$

$\Rightarrow t = 0.9$ s

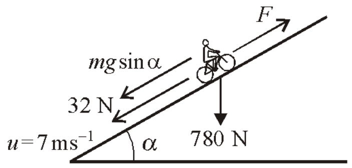
Using, $v = u - gt$

$v_P = -12$ ms^{-1} and $v_Q = -2$ ms^{-1}

Magnitude of velocities are, 12 m/s & 2 m/s

The direction of both is vert. downwards.

9.



(i) Applying Newton's 2nd law,

$F - (W\sin\alpha + 32) = ma \Rightarrow F = 102.8$ N

(ii) Using, $v^2 - u^2 = 2as$

$(0)^2 - (7)^2 = 2(-0.2)s \Rightarrow s = 122.5$ m

10. **(i)** By Newton's 2nd law, $-\mu R = ma$

$\Rightarrow -\mu mg = ma \Rightarrow a = -\mu g$. Therefore for P, $a = -2$ ms^{-2}. For Q, $a = -2.5$ ms^{-2}

Deceleration of P & Q are 2 ms^{-2} & 2.5 ms^{-2}

(ii) For P, $s_P = 8t + \frac{1}{2}(-2)t^2 \Rightarrow s = 8t - t^2$

For Q, $s_Q = 3t + \frac{1}{2}(-2.5)t^2 = 3t - 1.25t^2$

Now, $s_P - s_Q = 5$

$\Rightarrow 8t - t^2 - (3t - 1.25t^2) = 5$

$\Rightarrow 0.25t^2 + 5t - 5 = 0$,

by quad. formula, $t = 0.9544$

Using, $v = u + at$

$v_P = 6.09$ ms^{-1}, and $v_Q = 0.614$ ms^{-1}

11. **(i)** Distance travelled $= \frac{1}{2}(0.4)(28+19) = 9.4$ m

(ii) $a = 0.08$ ms^{-2}. Deceleration $= 0.1$ ms^{-2}

(iii) Total mass $= 800 + 100 = 900$ kg

Stage 1: Elevator accelerates upwards.

Applying Newton's 2nd law,

$T - mg = ma \Rightarrow T = 9072$ N

Stage 2: Elevator moves with constant speed.

$\therefore T = mg \Rightarrow T = 9000$ N

Stage 3: Elevator deccelerates and stop.

$\Rightarrow mg - T = ma \Rightarrow T = 8910$ N

(iv) Let R be the normal contact force exerted on the box by the floor. R is greatest when the elevator is moving up and is accelerating.

By 2nd law, $R - mg = ma \Rightarrow R = 1008$ N

Force R is least when the elevator is moving up and deccelerating. So, $mg - R = ma$

$\Rightarrow (100)(10) - R = (100)(0.1) \Rightarrow R = 990$ N

12. **(i)** $\sin\alpha = 0.28 \Rightarrow \cos\alpha = 0.96$

By 2nd law, $-mg\sin\alpha - \mu mg\cos\alpha = ma$

$\Rightarrow a = -g\sin\alpha - \mu g\cos\alpha$

$\Rightarrow a = -10(0.28) - \frac{1}{3}(10)(0.96) = -6$ ms^{-2}

(ii) Using, $v^2 - u^2 = 2as$

$0^2 - 5.4^2 = 2(-6)s \Rightarrow s = 2.43$ m

13. For upward motion, $v^2 - u^2 = 2gs$

$0 - 9^2 = 2(-10)s \Rightarrow s = 4.05$ m

Total distance $= 2(4.05) + 3.15$

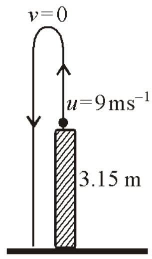
$= 11.25$ m

Using, $v = u + gt$, time $= 0.9$ s.

For downward motion, $s = \frac{1}{2}gt^2$

$3.15 + 4.05 = \frac{1}{2}(10)t^2 \Rightarrow t = 1.2$ s

Total time $= 1.2 + 0.9 = 2.1$ s



14. **(i)** Using, $v^2 - u^2 = 2gs$, the speed of P at the surface of the liquid is, $v = 12$ ms^{-1}

Inside the liquid, using, $v^2 - u^2 = 2as$

$(6)^2 - (12)^2 = 2a(0.8) \Rightarrow a = -67.5$

$\therefore$ deceleration $= 67.5$ ms^{-2}

By Newton's 2nd law, $R - mg = ma$

$\Rightarrow R = (0.2)(10) + (0.2)(67.5) = 15.5$ N

PROBABILITY & STATISTICS 1 (PAPER 5)

TOPIC 28

Representation of Data - Graphical Representations

1. The lengths of the diagonals in metres of the 9 most popular flat screen TVs and the 9 most popular conventional TVs are shown below.

Flat screen :	0.85	0.94	0.91	0.96	1.04	0.89	1.07	0.92	0.76
Conventional :	0.69	0.65	0.85	0.77	0.74	0.67	0.71	0.86	0.75

(i) Represent this information on a back-to-back stem-and-leaf diagram. [4]

(ii) Find the median and the interquartile range of the lengths of the diagonals of the 9 conventional TVs. [3]

(iii) Find the mean and standard deviation of the lengths of the diagonals of the 9 flat screen TVs. [2]

[J12/P61/Q5]

2. The back-to-back stem-and-leaf diagram shows the values taken by two variables *A* and *B*.

	A		*B*	
(3)	3 1 0	15	1 3 3 5	(4)
(2)	4 1	16	2 2 3 4 4 5 7 7 7 8	(10)
(3)	8 3 3	17	0 1 3 3 3 4 6 6 7 9 9	(11)
(12)	9 8 8 6 5 5 4 3 2 1 1 0	18	2 4 7	(3)
(8)	9 9 8 8 6 5 4 2	19	1 5	(2)
(5)	9 8 7 1 0	20	4	(1)

Key: 4 | 16 | 7 means $A = 0.164$ and $B = 0.167$.

(i) Find the median and the interquartile range for variable *A*. [3]

(ii) You are given that, for variable *B*, the median is 0.171, the upper quartile is 0.179 and the lower quartile is 0.164. Draw box-and-whisker plots for *A* and *B* in a single diagram on graph paper. [3]

[J12/P62/Q4]

3. Ashfaq and Kuljit have done a school statistics project on the prices of a particular model of head-phones for MP3 players. Ashfaq collected prices from 21 shops. Kuljit used the internet to collect prices from 163 websites.

(i) Name a suitable statistical diagram for Ashfaq to represent his data, together with a reason for choosing this particular diagram. [2]

(ii) Name a suitable statistical diagram for Kuljit to represent her data, together with a reason for choosing this particular diagram. [2]

[J12/P63/Q1]

4. Prices in dollars of 11 caravans in a showroom are as follows.

16 800 18 500 17 700 14 300 15 500 15 300 16 100 16 800 17 300 15 400 16 400

(i) Represent these prices by a stem-and-leaf diagram. [3]

(ii) Write down the lower quartile of the prices of the caravans in the showroom. [1]

(iii) 3 different caravans in the showroom are chosen at random and their prices are noted. Find the probability that 2 of these prices are more than the median and 1 is less than the lower quartile. [3]

[N12/P61/Q4]

5. The table summarises the times that 112 people took to travel to work on a particular day.

Time to travel to work (t minutes)	$0 < t \le 10$	$10 < t \le 15$	$15 < t \le 20$	$20 < t \le 25$	$25 < t \le 40$	$40 < t \le 60$
Frequency	19	12	28	22	18	13

(i) State which time interval in the table contains the median and which time interval contains the upper quartile. [2]

(ii) On graph paper, draw a histogram to represent the data. [4]

(iii) Calculate an estimate of the mean time to travel to work. [2]

[N12/P62/Q3]

6. In a survey, the percentage of meat in a certain type of take-away meal was found. The results, to the nearest integer, for 193 take-away meals are summarised in the table.

Percentage of meat	1 – 5	6 – 10	11 – 20	21 – 30	31 – 50
Frequency	59	67	38	18	11

(i) Calculate estimates of the mean and standard deviation of the percentage of meat in these take-away meals. [4]

(ii) Draw, on graph paper, a histogram to illustrate the information in the table. [5]

[N12/P63/Q4]

7. The following back-to-back stem-and-leaf diagram shows the annual salaries of a group of 39 females and 39 males.

	Females		Males	
(4)	5 2 0 0	20	3	(1)
(9)	9 8 8 7 6 4 0 0 0	21	0 0 7	(3)
(8)	8 7 5 3 3 1 0 0	22	0 0 4 5 6 6	(6)
(6)	6 4 2 1 0 0	23	0 0 2 3 3 5 6 7 7	(9)
(6)	7 5 4 0 0 0	24	0 1 1 2 5 5 6 8 8 9	(10)
(4)	9 5 0 0	25	3 4 5 7 7 8 9	(7)
(2)	5 0	26	0 4 6	(3)

Key: 2 | 20 | 3 means \$20 200 for females and \$20 300 for males.

(i) Find the median and the quartiles of the females' salaries. [2]

You are given that the median salary of the males is \$24 000, the lower quartile is \$22 600 and the upper quartile is \$25 300.

(ii) Represent the data by means of a pair of box-and-whisker plots in a single diagram on graph paper. [3]

[J13/P61/Q3]

ANSWERS

Topic 28 - Representation of Data - Graphical Representations

1. **(i)**

Flat screen		Conventional
	6	5 7 9
6	7	1 4 5 7
9 5	8	5 6
6 4 2 1	9	
7 4	10	

Key: 5 | 8 | 4

Means 0.84 m for conventional
0.85 m for full screen.

(ii) Median = 0.74 m

Interquartile range = 0.81 − 0.68 = 0.13 m

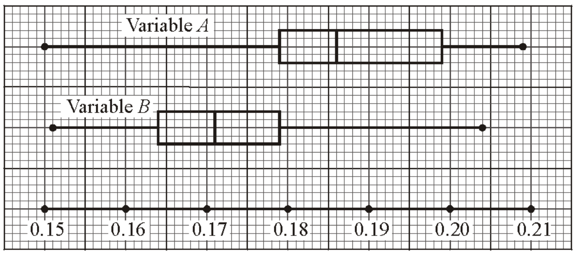
(iii) Mean $= \bar{x} = \dfrac{\sum x}{n} = 0.927$ m

$S.D = \sqrt{\dfrac{\sum x^2}{n} - (\bar{x})^2} = 0.0882$ m

2. **(i)** Median for variable $A = 0.186$

Interquartile range $= Q_3 - Q_1 = 0.019$

(ii)



3. **(i)** Stem and leaf .

Reason: The given data consists of lesser number of values.

(ii) Histogram or cumulative frequency graph.

Reason: The given data consists of large number of values.

4. **(i)**

14	3
15	3 4 5
16	1 4 8 8
17	3 7
18	5

Key : 16 | 1

Means \$16 100.

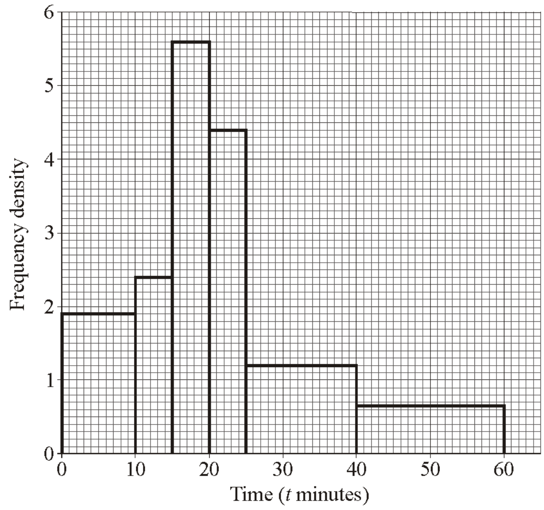
(ii) Lower quartile = \$15 400

(iii) Probability $= 3\left(\dfrac{2}{11} \times \dfrac{5}{10} \times \dfrac{4}{9}\right) = \dfrac{4}{33}$

5. **(i)** Median lies in the interval $15 < t \le 20$

Upper quartile lies in interval $25 < t \le 40$

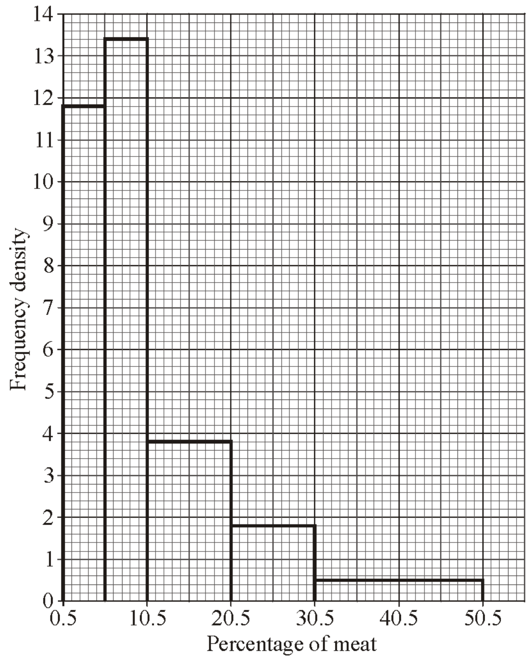
(ii)



(iii) Mean $= \bar{t} = \dfrac{\sum ft}{\sum f} = \dfrac{2465}{112} = 22.0$ minutes

6. **(i)** Mean = 11.4, $S.D = \sqrt{\dfrac{\sum fx^2}{\sum f} - (\bar{x})^2} = 9.79$

(ii)

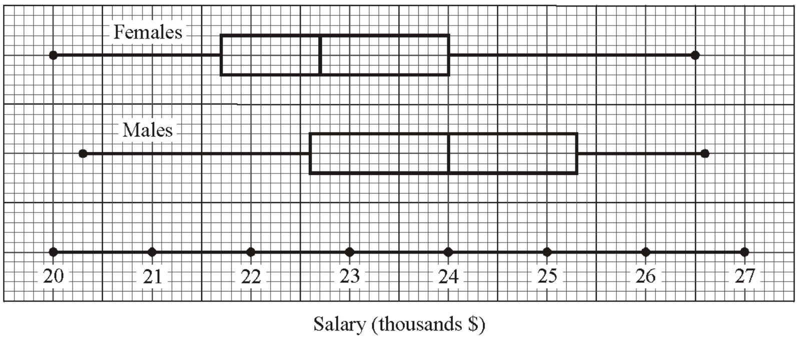


7. **(i)** Median = \$22 700

Lower quartile = \$21 700

Upper quartile = \$24 000

(ii)



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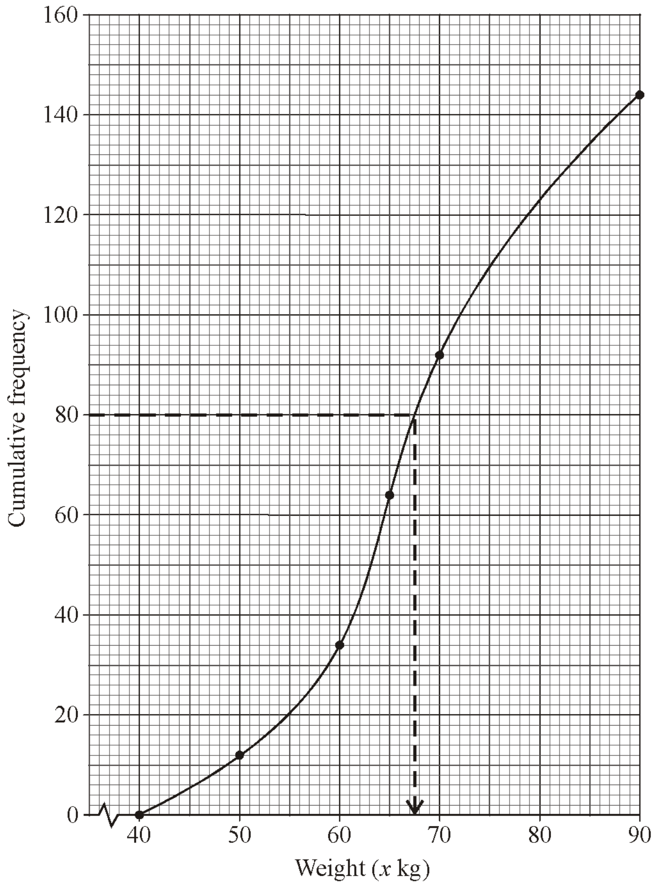
8. **(i)** Refer to graph

(ii) $144 - 64 = 80$ people.

So from graph, $c = 67.5$ kg

(iii) Mean $= \frac{\sum fx}{\sum f} = \frac{9675}{144} = 67.2$ kg

$S.D = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2} = 11.3$ kg



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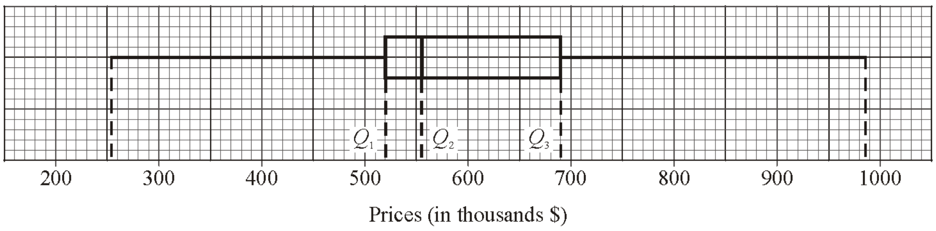
9. **(i)**



(ii) Higher outlier $= Q_3 + 1.5(IQ) = 945\,000$. Prices of expensive houses are: 957 000, 986 000

(iii) Box & wisker diagram does not show all values.