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# IGCSE

(Syllabus 0580)

# MATHEMATICS

## Paper 2 (Extended) - All Variants

## (Topical)

| | |
|---|---|
| period | 2018 to June-2024 |
| contents | June & November, Paper 2 (P21, P22 & P23) Worked Solutions |
| form | Topic By Topic |
| compiled for | IGCSE |



Tel No : 042-35201010
Mobile No : 0300-8447654
E-Mail : info@redspot.com.pk
Website : www.redspot.pk
Address : P.O. Box 5041, Model Town, Lahore, Pakistan.

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# CONTENTS

# TOPIC 1

# Numbers

**1.** Calculate.

$$\frac{5.38 - 0.98}{0.743 - 0.343}$$

.....[1]

*[Nov/2018/P23/Q4][Note: Question is modified to solve without a calculator]*

---

**2.** **Without using a calculator**, work out $\frac{1}{4} \div \frac{2}{3}$.

You must show all your working and give your answer as a fraction.

..... [2]

*[Nov/2018/P23/Q9]*

---

**3.** Change the recurring decimal 0.18 to a fraction.
You must show all your working.

..... [2]

*[Nov/2018/P23/Q13]*

---

**4.** Calculate $\sqrt[3]{8.2^2 - 3.24}$

..... [1]

*[June/2019/P21/Q3][Note: Question is modified to solve without a calculator]*

---

**5.** Write the recurring decimal $0.4\dot{7}$ as a fraction.
Show all your working.

..... [2]

*[June/2019/P21/Q9]*

---

**6.** 27 28 29 30 31 32 33

From the list of numbers, write down

(a) a multiple of 7,

..... [1]

(b) a cube number,

..... [1]

(c) a prime number.

..... [1]

*[June/2019/P21/Q12]*

---

**7.** Work out $\frac{5}{6}+\frac{2}{3}$. You must show all your working and give your answer as a mixed number in its simplest form.

..... [3]

*[June/2019/P21/Q14]*

---

**8.** Write down a prime number between 50 and 60.

..... [1]

*[June/2019/P22/Q1]*

---

**9.** Write the recurring decimal $0.\dot{7}$ as a fraction.

..... [1]

*[June/2019/P22/Q3]*

---

**10.** Find the highest common factor (HCF) of 90 and 48.

..... [2]

*[June/2019/P22/Q9]*

---

**11.** Work out $2\frac{1}{4} \div \frac{3}{7}$. You must show all your working and give your answer as a mixed number in its simplest form.

..... [3]

*[June/2019/P22/Q13]*

---

**12.** Calculate.

(a) $-12 \div -2$

..... [1]

(b) $\sqrt[3]{2^4 + 11}$

..... [1]

*[June/2019/P23/Q6] [Note: Part (b) is modified to solve without a calculator]*

---

**13.** Here is a list of numbers.

$$21 \qquad \frac{2}{3} \qquad \sqrt{13} \qquad 31 \qquad \sqrt{121} \qquad 51 \qquad 0.7$$

From this list, write down

(a) a prime number,

..... [1]

(b) an irrational number.

..... [1]

*[June/2019/P23/Q7]*

---

**14.** Work out $\frac{12}{35} \times \frac{7}{9}$. You must show all your working and give your answer as a fraction in its simplest form.

..... [2]

*[June/2019/P23/Q9]*

---

**15.** Calculate. $\dfrac{16-3.4}{4.2}\times 1.2$

Give your answer correct to 2 significant figures.

..... [2]

*[Nov/2019/P21/Q3] [Note: Question is modified to solve without a calculator]*

---

**16.** Work out $\dfrac{5}{16}\times 1\dfrac{1}{7}$. You must show all your working and give your answer as a fraction in its simplest form.

..... [2]

*[Nov/2019/P21/Q8]*

---

**17.** Calculate $\dfrac{5}{8}+\sqrt[3]{216}$. Give your answer as a fraction

..... [1]

*[Nov/2019/P22/Q2] [Note: Question is modified to solve without a calculator]*

---

**18.** Write the recurring decimal $0.6\dot{7}$ as a fraction.
Show all your working and give your answer in its simplest form.

..... [2]

*[Nov/2019/P22/Q10]*

---

**19.** Work out $3\dfrac{5}{8}-1\dfrac{2}{3}$. You must show all your working and give your answer as a mixed number in its simplest form.

..... [3]

*[Nov/2019/P22/Q11]*

---

**20.** Calculate $\sqrt{144^{0.5}+4\times6}$

..... [1]

*[Nov/2019/P23/Q2] [Note: Question is modified to solve without a calculator]*

---

**21.** Here is a list of numbers.

87 77 57 47 27

From this list, write down

(a) a cube number,

..... [1]

(b) a prime number.

..... [1]

*[Nov/2019/P23/Q3]*

---

**22.** Find the highest common factor (HCF) of 84 and 105.

..... [2]

*[Nov/2019/P23/Q4]*

---

**23.** Work out $\frac{2}{3}+\frac{1}{4}\times\frac{2}{3}$.

Write down all the steps of your working and give your answer as a fraction in its simplest form.

..... [4]

*[Nov/2019/P23/Q15]*

---

**24.** Find the highest **odd** number that is a factor of 60 and a factor of 90.

..... [1]

*[June/2020/P21/Q2]*

---

**25.** $234 = 2\times3^2\times13$ $1872 = 2^4\times3^2\times13$ $234\times1872 = 438\,048$

Use this information to write 438 048 as a product of its prime factors.

..... [1]

*[June/2020/P21/Q7]*

---

**26.** Work out $\left(2\frac{1}{3}-\frac{7}{8}\right)\times\frac{6}{25}$.

You must show all your working and give your answer as a fraction in its simplest form.

..... [4]

*[June/2020/P21/Q8]*

---

**27.** Write down

(a) a square number greater than 10,

..... [1]

(b) an irrational number.

..... [1]

*[June/2020/P22/Q4]*

---

**28.** Write $2^{-4}$ as a fraction.

..... [1]

*[June/2020/P22/Q8] [Note: Question is modified to solve without a calculator]*

---

**29.** Work out $1\frac{3}{4}-\frac{11}{12}$.

You must show all your working and give your answer as a fraction in its simplest form.

..... [3]

*[June/2020/P22/Q11]*

---

**30.** 32 33 34 35 36 37 38 39

From this list of numbers, write down

(a) a multiple of 8,

..... [1]

(b) a square number,

..... [1]

(c) a prime number.

..... [1]

*[June/2020/P23/Q1]*

---

**31.** Work out $3\frac{1}{4} - 2\frac{2}{3}$.

You must show all your working and give your answer as a fraction in its simplest form.

..... [3]

*[June/2020/P23/Q7]*

---

**32.** Work out $2\frac{2}{3} \times 2\frac{3}{4}$ .

You must show all your working and give your answer as a mixed number in its simplest form.

..... [3]

*[Nov/2020/P21/Q6]*

---

**33.** (a) 1, 2, 3, 5 and 7 are all common factors of two numbers.

Write down the digit that the two numbers must end in.

..... [1]

(b) Write 84 as a product of its prime factors.

..... [2]

*[Nov/2020/P21/Q8]*

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# SOLUTIONS

## Topic 1 - Numbers

**1.** $\dfrac{5.38-0.98}{0.743-0.343}$

$=\dfrac{4.4}{0.4}\times\dfrac{10}{10}$

$=\dfrac{44}{4}=11$

**2.** $\dfrac{1}{4}\div\dfrac{2}{3}$

$=\dfrac{1}{4}\times\dfrac{3}{2}=\dfrac{3}{8}$

**3.** Let $x=0.1\dot{8}$

$\Rightarrow \quad x=0.188888$

Multiply by 10,

$\Rightarrow \quad 10x=1.88888$ ..... (1)

Multiply by 100,

$\Rightarrow \quad 100x=18.8888$ ..... (2)

Subtract (1) from (2)

$100x=18.8888...$

$10x=\ 1.8888...$

$90x=17$

$\Rightarrow \quad x=\dfrac{17}{90}$

**4.** $\sqrt[3]{8.2^2-3.24}$

$=\sqrt[3]{67.24-3.24}$

$=\sqrt[3]{64}=\sqrt[3]{4^3}=4$

**5.** Let $x=0.4\dot{7}$

$\Rightarrow \quad x=0.477777$

Multiply by 10,

$\Rightarrow \quad 10x=4.77777....$ (1)

Multiply by 100,

$\Rightarrow \quad 100x=47.77777....$ (2)

Subtract (1) from (2)

$100x=47.77777...$

$10x=\ 4.77777...$

$90x=43$

$\Rightarrow \quad x=\dfrac{43}{90}$

**6.** **(a)** Multiple of 7 = 28

**(b)** Cube number = 27

**(c)** Prime number = 29 or 31

**7.** $\dfrac{5}{6}+\dfrac{2}{3}$

$=\dfrac{5+4}{6}$

$=\dfrac{9}{6}=\dfrac{3}{2}=1\dfrac{1}{2}$

**8.** 53 or 59

**9.** Let $x=0.\dot{7}$

$\Rightarrow \quad x=0.77777....$ (1)

$\Rightarrow \quad 10x=7.77777....$ (2)

Subtract (1) from (2)

$10x=7.77777...$

$x=0.77777...$

$9x=7$

$\Rightarrow \quad x=\dfrac{7}{9}$

**10.** $90=2\times3\times3\times5$

$48=2\times2\times2\times2\times3$

$\therefore$ HCF $=2\times3=6$

**11.** $2\dfrac{1}{4}\div\dfrac{3}{7}$

$=\dfrac{9}{4}\times\dfrac{7}{3}$

$=\dfrac{21}{4}=5\dfrac{1}{4}$

**12.** **(a)** $-12\div-2$

$=\dfrac{-12}{-2}=6$

**(b)** $\sqrt[3]{2^4+11}$

$=\sqrt[3]{16+11}$

$=\sqrt[3]{27}\ =\sqrt[3]{3^3}=3$

**13.** **(a)** Prime number = 31 or $\sqrt{121}$

**(b)** Irrational number = $\sqrt{13}$

**14.** $\frac{12}{35}\times\frac{7}{9}$

$=\frac{4}{5}\times\frac{1}{3}=\frac{4}{15}$

**15.** $\frac{16-3.4}{4.2}\times 1.2$

$=\frac{12.6}{4.2}\times 1.2$

$=\frac{126}{42}\times 1.2$

$=3\times 1.2=3.6$

**16.** $\frac{5}{16}\times 1\frac{1}{7}$

$=\frac{5}{16}\times\frac{8}{7}=\frac{5}{14}$

**17.** $\frac{5}{8}+\sqrt[3]{216}$

$=\frac{5}{8}+\sqrt[3]{6^3}$

$=\frac{5}{8}+6=\frac{53}{8}$

**18.** Let $x=0.6\dot{7}$

$\Rightarrow \quad x=0.677777...$ (1)

Multiply (1) by 10,

$\Rightarrow \quad 10x=6.77777...$ (2)

Multiply (1) by 100,

$\Rightarrow \quad 100x=67.77777....$ (3)

Subtract (2) from (3)

$$\begin{array}{r} 100x=67.77777... \\ \underline{10x=\ 6.77777...} \\ 90x\ =61 \end{array}$$

$\Rightarrow \quad x=\frac{61}{90}$

**19.** $3\frac{5}{8}-1\frac{2}{3}$

$=\frac{29}{8}-\frac{5}{3}$

$=\frac{87-40}{24}$

$=\frac{47}{24}=1\frac{23}{24}$

**20.** $\sqrt{144^{0.5}+4\times 6}$

$=\sqrt{(12^2)^{0.5}+24}$

$=\sqrt{12+24}$

$=\sqrt{36}=\pm 6$

**21.** **(a)** Cube number = 27

**(b)** Prime number = 47

**22.** $84=2^2\times 3\times 7$

$105=3\times 5\times 7$

$\therefore$ H.C.F. $=3\times 7=21$

**23.** $\frac{2}{3}+\frac{1}{4}\times\frac{2}{3}$

$=\frac{2}{3}+\frac{2}{12}$

$=\frac{2}{3}+\frac{1}{6}$

$=\frac{4+1}{6}=\frac{5}{6}$

**24.** $60=2\times 2\times 3\times 5$

$90=2\times 3\times 3\times 5$

$\therefore$ Highest odd number that is factor of 60 and $90=3\times 5=15$

**25.** $234=2\times 3^2\times 13$

$1872=2^4\times 3^2\times 13$

$\therefore \quad 438048=(2\times 3^2\times 13)\times(2^4\times 3^2\times 13)$

$=2^5\times 3^4\times 13^2$

**26.** $\left(2\frac{1}{3}-\frac{7}{8}\right)\times\frac{6}{25}$

$=\left(\frac{7}{3}-\frac{7}{8}\right)\times\frac{6}{25}$

$=\frac{56-21}{24}\times\frac{6}{25}$

$=\frac{35}{24}\times\frac{6}{25}=\frac{7}{20}$

**27.** **(a)** 16 **(b)** $\sqrt{3}$

**28.** $2^{-4}=\frac{1}{2^4}$

$=\frac{1}{16}$

**29.** $1\frac{3}{4}-\frac{11}{12}$

$=\frac{7}{4}-\frac{11}{12}$

$=\frac{21-11}{12}=\frac{10}{12}=\frac{5}{6}$

**30.** **(a)** Multiple of 8 = 32

**(b)** Square number = 36

**(c)** Prime number = 37

**31.** $3\frac{1}{4}-2\frac{2}{3}$

$=\frac{13}{4}-\frac{8}{3}$

$=\frac{39-32}{12}=\frac{7}{12}$

**32.** $2\frac{2}{3}\times 2\frac{3}{4}$

$=\frac{8}{3}\times\frac{11}{4}$

$=\frac{22}{3}=7\frac{1}{3}$

**33.** **(a)** The digit that two numbers must end in = 0

> Note : Since 2 is a factor, the last digit must be 0, 2, 4, 6, or 8
> Since 5 is a factor, the last digit must be 0 or 5
> Since both 2 and 5 are factors, the last digit must be 0.

**(b)** $84=2\times 2\times 3\times 7$

**34.** Let $x=0.1\dot{7}$

$\Rightarrow \quad x=0.177777...$ (1)

Multiply (1) by 10,

$\Rightarrow \quad 10x=1.77777...$ (2)

Multiply (1) by 100,

$\Rightarrow \quad 100x=17.7777...$ (3)

Subtract (2) from (3)

$$\begin{array}{r} 100x=17.7777... \\ \underline{10x=\ 1.7777...} \\ 90x\ =16 \end{array}$$

$\Rightarrow \quad x=\frac{16}{90}=\frac{8}{45}$

**35.** 200017

**36.** $7-(5-3)+4=9$

**37.** $\frac{5}{6}\div 1\frac{1}{3}$

$=\frac{5}{6}\div\frac{4}{3}$

$=\frac{5}{6}\times\frac{3}{4}=\frac{5}{8}$

**38.** 64

**39.** $\frac{4}{\sqrt{0.0025}}$

$=\frac{4}{\sqrt{\frac{25}{10000}}}$

$=\frac{4}{\sqrt{\left(\frac{5}{100}\right)^2}}$

$=\frac{4}{\frac{5}{100}}=4\times\frac{100}{5}=80$

**40.** $1\frac{1}{7}\times 2\frac{1}{10}$

$=\frac{8}{7}\times\frac{21}{10}$

$=\frac{12}{5}=2\frac{2}{5}$

**41.** $N=2^4\times 3\times 7^5$

$\Rightarrow \quad (3\times 7)N=2^4\times 3\times 7^5\times(3\times 7)$

$\Rightarrow \quad 21N=2^4\times 3^2\times 7^6$

$\therefore \quad P=21$

**42.** $1\frac{3}{8}-\frac{5}{6}$

$=\frac{11}{8}-\frac{5}{6}$

$=\frac{33-20}{24}=\frac{13}{24}$

**43.** $\sqrt[4]{0.0256}$

$=\left(\frac{256}{10000}\right)^{\frac{1}{4}}$

$=\left(\left(\frac{4}{10}\right)^4\right)^{\frac{1}{4}}=\frac{4}{10}=0.4$

**44.** **(a)** $\frac{1}{0.2}=\frac{10}{2}=5$

$\therefore$ Reciprocal of 0.2 is 5

Prime number between 90 and 100 is 97

**(b)** $\sqrt{7}$

**45.** $\frac{2}{3}\div 1\frac{3}{7}$

$=\frac{2}{3}\div\frac{10}{7}$

$=\frac{2}{3}\times\frac{7}{10}=\frac{7}{15}$

# TOPIC 14

# Solutions of Equations

**1.** Solve the equation. $9f + 11 = 3f + 23$

$f$ = ..... [2]

*[June/2019/P22/Q6]*

---

**2.** Solve the simultaneous equations. You must show all your working.

$$5x + 8y = 4$$
$$\frac{1}{2}x + 3y = 7$$

$x$ = .....

$y$ = .....

[3]

*[June/2019/P22/Q14]*

---

**3.** Rearrange this formula to make $m$ the subject.

$$P = \frac{k + m}{m}$$

..... [4]

*[June/2019/P22/Q19]*

---

**4.** Rearrange $2(w+h)=P$ to make $w$ the subject.

$w$ = ..... [2]

*[June/2019/P23/Q10]*

---

**5.** Complete this statement with an expression in terms of $m$.

$$18m^3+9m^2+14m+7 \;=\; (9m^2+7)(.....)$$

[2]

*[June/2019/P23/Q11]*

---

**6.** One solution of the equation $ax^2+a=150$ is $x=7$.

(a) Find the value of $a$.

$a$ = ..... [2]

(b) Find the other solution.

$x$ = ..... [1]

*[June/2019/P23/Q14]*

---

**7.** Solve. $\frac{x-2}{3}=3$

$x$ = ..... [2]

*[Nov/2019/P21/Q6]*

---

**8.** $P=2r+\pi r$

Rearrange the formula to write $r$ in terms of $P$ and $\pi$.

$r$ = ..... [2]

*[Nov/2019/P21/Q11]*

---

**9.** $y=mx+c$

Find the value of $y$ when $m = –3$, $x = –2$ and $c = –8$.

$y$ = ..... [2]

*[June/2020/P22/Q5]*

---

**10.** Solve the equation. $\frac{1-x}{3}=5$

$x$ = ..... [2]

*[June/2020/P22/Q14]*

---

**11.** Make $y$ the subject of the formula, $h^2=x^2+2y^2$

$y$ = ..... [3]

*[June/2020/P22/Q19]*

---

**12.** (a) Write $x^2 - 18x - 40$ in the form $(x+k)^2 + h$.

..... [2]

(b) Use your answer to **part (a)** to solve the equation $x^2 - 18x - 40 = 0$

$x$ = ..... or $x$ = ..... [2]

*[June/2020/P23/Q18] [Note: Question is modified to solve without a calculator]*

---

**13.** Make $x$ the subject of this formula.

$$2y = 5x - 7$$

$x$ = ..... [2]

*[Nov/2020/P21/Q7]*

---

**14.** Solve the equation. $6 - 2x = 3x$

$x$ = ..... [2]

*[Nov/2020/P22/Q3]*

---

**15.** Solve the simultaneous equations.

$$2x + y = 7$$
$$3x - y = 8$$

$x$ = .....

$y$ = .....

[2]

*[Nov/2020/P22/Q9]*

---

**16.** Solve the simultaneous equations. You must show all your working.

$$3x - 8y = 22$$
$$x + 4y = 4$$

$x$ = .....

$y$ = .....

[3]

*[Nov/2020/P23/Q10]*

---

**17.** $m = 2p + \sqrt{\frac{x}{y}}$

Make $x$ the subject of this formula.

$x$ = ..... [3]

*[Nov/2020/P23/Q15]*

**18.** Solve the simultaneous equations. You must show all your working.

$$2x + y = 3$$
$$x - 5y = 40$$

$x$ = .....

$y$ = .....

[3]

*[June/2021/P21/Q7]*

---

**19.** Make $h$ the subject of the formula. $2mh = g(1 - h)$.

$h$ = ..... [4]

*[June/2021/P21/Q15]*

---

**20.** $a = \dfrac{b^2}{5c}$

Find $b$ when $a = 4.9$ and $c = 2$.

$b$ = ..... [2]

*[June/2021/P22/Q8] [Note: Question is modified to solve without a calculator]*

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# SOLUTIONS

## Topic 14 - Solutions of Equations

**1.** $9f+11=3f+23$

$9f-3f=23-11$

$6f=12 \Rightarrow f=\frac{12}{6}=2$

**2.** $5x+8y=4$ ..... (1)

$\frac{1}{2}x+3y=7$ ..... (2)

Solving the equations simultaneously,

eq. (1): $5x+8y=4$

eq. (2)$\times 10$: $5x+30y=70$

$-22y=-66$

$\Rightarrow y=\frac{-66}{-22}=3$

Substitute $y=3$ into eq. (1)

$5x+8(3)=4$

$\Rightarrow 5x+24=4 \Rightarrow 5x=-20 \Rightarrow x=-4$

$\therefore x=-4, \quad y=3$

**3.** $P=\frac{k+m}{m}$

$Pm=k+m$

$Pm-m=k$

$m(P-1)=k$

$m=\frac{k}{P-1}$

**4.** $2(w+h)=P$

$w+h=\frac{P}{2} \Rightarrow w=\frac{P}{2}-h$

**5.** $\text{LHS}=18m^3+9m^2+14m+7$

$=18m^3+14m+9m^2+7$

$=2m(9m^2+7)+1(9m^2+7)$

$=(9m^2+7)(2m+1)$

$\therefore$ Required expression is, $2m+1$

**6.** **(a)** $ax^2+a=150$

Substitute $x=7$,

$\Rightarrow a(7)^2+a=150$

$\Rightarrow 49a+a=150$

$\Rightarrow 50a=150 \Rightarrow a=\frac{150}{50}=3$

**(b)** $ax^2+a=150$

Substitute $a=3$,

$\Rightarrow 3x^2+3=150$

$\Rightarrow 3x^2=147$

$\Rightarrow x^2=\frac{147}{3}$

$\Rightarrow x^2=49 \Rightarrow x=\pm 7$

$\therefore$ Other solution is, $x=-7$

**7.** $\frac{x-2}{3}=3$

$x-2=9$

$x=9+2=11$

**8.** $P=2r+\pi r$

$\Rightarrow 2r+\pi r=P$

$\Rightarrow r(2+\pi)=P \Rightarrow r=\frac{P}{2+\pi}$

**9.** $y=mx+c$

$y=(-3)(-2)+(-8)$

$y=6-8=-2$

**10.** $\frac{1-x}{3}=5$

$1-x=15$

$-x=14 \Rightarrow x=-14$

**11.** $h^2=x^2+2y^2$

$\Rightarrow x^2+2y^2=h^2$

$\Rightarrow 2y^2=h^2-x^2$

$\Rightarrow y^2=\frac{h^2-x^2}{2} \Rightarrow y=\pm\sqrt{\frac{h^2-x^2}{2}}$

**12.** **(a)** $x^2-18x-40$

Applying completing the square method,

$=x^2-2(x)(9)+(9)^2-(9)^2-40$

$=(x-9)^2-81-40$

$=(x-9)^2-121$

**(b)** $x^2 - 18x - 40 = 0$

$\Rightarrow \quad (x-9)^2 - 121 = 0$

$\Rightarrow \quad (x-9)^2 = 121$

$\Rightarrow \quad x - 9 = \pm\sqrt{121}$

$\Rightarrow \quad x - 9 = \pm 11$

$\Rightarrow \quad x - 9 = 11 \quad \text{or} \quad x - 9 = -11$

$\therefore \quad x = 20 \text{ or } x = -2$

**13.** $2y = 5x - 7$

$\Rightarrow \quad 5x = 2y + 7 \quad \Rightarrow \quad x = \dfrac{2y+7}{5}$

**14.** $6 - 2x = 3x$

$6 = 3x + 2x$

$6 = 5x \quad \Rightarrow \quad x = \dfrac{6}{5} = 1.2$

**15.** $2x + y = 7 \quad \Rightarrow \quad y = 7 - 2x$ ..... (1)

$3x - y = 8$ ..... (2)

Substitute eq. (1) into eq. (2),

$3x - (7 - 2x) = 8$

$\Rightarrow \quad 3x - 7 + 2x = 8$

$\Rightarrow \quad 5x = 15 \quad \Rightarrow \quad x = 3$

Substitute $x = 3$ into eq. (1), $y = 7 - 2(3) = 1$

$\therefore \quad x = 3, \quad y = 1$

**16.** $3x - 8y = 22$ ..... (1)

$x + 4y = 4 \quad \Rightarrow \quad x = 4 - 4y$ ..... (2)

Substitute eq. (2) into eq. (1),

$3(4 - 4y) - 8y = 22$

$\Rightarrow \quad 12 - 12y - 8y = 22$

$\Rightarrow \quad -20y = 10 \quad \Rightarrow \quad y = \dfrac{10}{-20} = -\dfrac{1}{2}$

Substitute $y = -\dfrac{1}{2}$ into eq. (2),

$x = 4 - 4\left(-\dfrac{1}{2}\right) \quad \Rightarrow \quad x = 4 + 2 = 6$

$\therefore \quad x = 6, \quad y = -\dfrac{1}{2}$

**17.** $m = 2p + \sqrt{\dfrac{x}{y}}$

$\Rightarrow \quad \sqrt{\dfrac{x}{y}} = m - 2p$

$\Rightarrow \quad \dfrac{x}{y} = (m - 2p)^2$

$\Rightarrow \quad x = y(m - 2p)^2$

**18.** $2x + y = 3$ ..... (1)

$x - 5y = 40$ ..... (2)

Solving the equations simultaneously,

eq. (1)×5 : $10x + 5y = 15$

eq. (2) : $x - 5y = 40$

$11x = 55$

$\Rightarrow \quad x = \dfrac{55}{11} = 5$

Substitute $x = 5$ into eq. (1)

$2(5) + y = 3 \quad \Rightarrow \quad 10 + y = 3 \quad \Rightarrow \quad y = -7$

$\therefore \quad x = 5, \quad y = -7$

**19.** $2mh = g(1 - h)$

$\Rightarrow \quad 2mh = g - gh$

$\Rightarrow \quad 2mh + gh = g$

$\Rightarrow \quad h(2m + g) = g \quad \Rightarrow \quad h = \dfrac{g}{2m+g}$

**20.** $a = \dfrac{b^2}{5c}$

$\Rightarrow \quad 4.9 = \dfrac{b^2}{5(2)}$

$\Rightarrow \quad \dfrac{b^2}{10} = 4.9$

$\Rightarrow \quad b^2 = 49$

$\Rightarrow \quad b = \pm\sqrt{49} = \pm 7$

**21.** $4x - 2y = -13$ ..... (1)

$-3x + 4y = 11$ ..... (2)

Solving the equations simultaneously,

eq. (1)×2 : $8x - 4y = -26$

eq. (2) : $-3x + 4y = 11$

$5x = -15$

$\Rightarrow \quad x = \dfrac{-15}{5} = -3$

Substitute $x = -3$ into eq. (2)

$-3(-3) + 4y = 11$

$\Rightarrow \quad 9 + 4y = 11 \quad \Rightarrow \quad 4y = 2 \quad \Rightarrow \quad y = \dfrac{1}{2}$

$\therefore \quad x = -3, \quad y = \dfrac{1}{2}$

**22.** $y = \dfrac{3x - 2}{1 - x}$

$\Rightarrow \quad y(1 - x) = 3x - 2$

$\Rightarrow \quad y - xy = 3x - 2$

$\Rightarrow \quad y + 2 = 3x + xy$

$\Rightarrow \quad y + 2 = x(3 + y) \quad \Rightarrow \quad x = \dfrac{y+2}{3+y}$

**23.** $(5x-3)(2x+7)=0$

$\Rightarrow \quad 5x-3=0 \quad \text{or} \quad 2x+7=0$

$\Rightarrow \quad 5x=3 \quad \text{or} \quad 2x=-7$

$\therefore \quad x=\frac{3}{5} \quad \text{or} \quad x=-\frac{7}{2}$

**24.** $y=x^2-9x+21$ ..... (1)

$y=2x-3$ ..... (2)

Substitute eq. (1) into eq. (2),

$x^2-9x+21=2x-3$

$\Rightarrow \quad x^2-9x+21-2x+3=0$

$\Rightarrow \quad x^2-11x+24=0$

$\Rightarrow \quad x^2-8x-3x+24=0$

$\Rightarrow \quad x(x-8)-3(x-8)=0$

$\Rightarrow \quad (x-8)(x-3)=0$

$\Rightarrow \quad x-8=0 \quad \text{or} \quad x-3=0$

$\Rightarrow \quad x=8 \quad \text{or} \quad x=3$

Substitute $x=8$ into eq. (2),

$y=2(8)-3 \quad \Rightarrow \quad y=16-3=13$

Substitute $x=3$ into eq. (2),

$y=2(3)-3 \quad \Rightarrow \quad y=6-3=3$

$\therefore \quad x=8, \quad y=13$

$x=3, \quad y=3$

**25.** **(a)** $P=M(g^2+h^2)$

$\Rightarrow \quad P=100(3^2+4^2)$

$\Rightarrow \quad P=100(9+16)$

$=100(25)=2500$

**(b)** $P=M(g^2+h^2)$

$\Rightarrow \quad P=Mg^2+Mh^2$

$\Rightarrow \quad Mg^2=P-Mh^2$

$\Rightarrow \quad g^2=\frac{P-Mh^2}{M} \quad \Rightarrow \quad g=\pm\sqrt{\frac{P-Mh^2}{M}}$

**26.** $y=x^2-3x-13$ ..... (1)

$y=x-1$ ..... (2)

Solving the two equations simultaneously,

$\Rightarrow \quad x^2-3x-13=x-1$

$\Rightarrow \quad x^2-3x-13-x+1=0$

$\Rightarrow \quad x^2-4x-12=0$

$\Rightarrow \quad x^2-6x+2x-12=0$

$\Rightarrow \quad x(x-6)+2(x-6)=0$

$\Rightarrow \quad (x-6)(x+2)=0$

$\Rightarrow \quad x-6=0 \quad \text{or} \quad x+2=0$

$\Rightarrow \quad x=6 \quad \text{or} \quad x=-2$

Substitute $x=6$ into eq. (2), $\quad y=6-1=5$

Substitute $x=-2$ into eq. (2), $\quad y=-2-1=-3$

$\therefore \quad x=6, \quad y=5$

$x=-2, \quad y=-3$

**27.** **(a)** $s=\frac{1}{2}at^2$

$\Rightarrow \quad s=\frac{1}{2}(0.9)(4)^2$

$=\frac{1}{2}(0.9)(16)$

$=(0.9)(8)=7.2$

**(b)** $s=\frac{1}{2}at^2$

$\Rightarrow \quad 2s=at^2$

$\Rightarrow \quad t^2=\frac{2s}{a} \quad \Rightarrow \quad t=\pm\sqrt{\frac{2s}{a}}$

**28.** **(a)** L.H.S. $=x^2+8x+10$

Using completing the square method,

$=x^2+2(x)(4)+(4)^2-(4)^2+10$

$=(x+4)^2-16+10$

$=(x+4)^2-6$

$\therefore \quad p=4, \quad q=-6$

**(b)** $x^2+8x+10=30$

Using the result of part (a),

$\Rightarrow \quad (x+4)^2-6=30$

$\Rightarrow \quad (x+4)^2=36$

$\Rightarrow \quad x+4=\pm 6$

$\Rightarrow \quad x+4=6 \quad \text{or} \quad x+4=-6$

$\therefore \quad x=2 \quad \text{or} \quad x=-10$

**29.** $x-3y=7 \quad \Rightarrow \quad x=7+3y$ ..... (1)

$2x-3y=11$ ..... (2)

Substitute eq. (1) into eq. (2),

$2(7+3y)-3y=11$

$\Rightarrow \quad 14+6y-3y=11$

$\Rightarrow \quad 3y=-3 \quad \Rightarrow \quad y=-1$

Substitute $y=-1$ into eq. (1), $\quad x=7+3(-1)=4$

$\therefore \quad x=4, \quad y=-1$

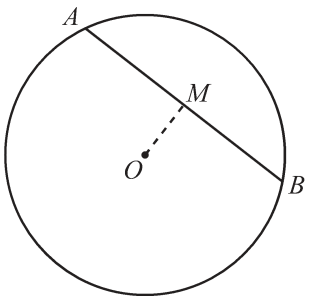
# TOPIC 27

# Circle Theorems

**1.** The diagram shows a circle, centre $O$.

$AB$ is a chord of length 12 cm.
$M$ is the mid-point of $AB$ and $OM = 4$ cm.

Calculate the radius of the circle. Give your answer in the form $\sqrt{q}$.




..... cm [3]

*[June/2018/P22/Q16] [Note: Question is modified to solve without a calculator]*

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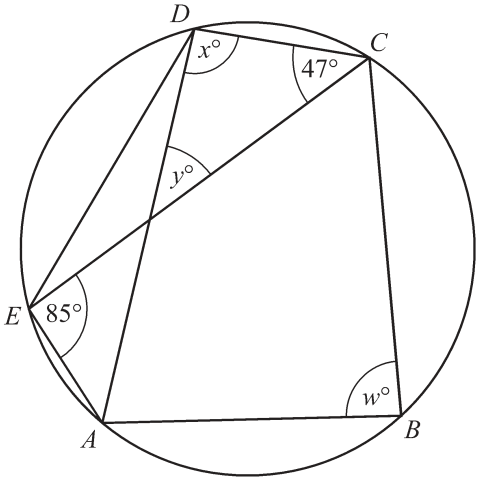
**2.** The points $A$, $B$, $C$, $D$ and $E$ lie on the circumference of the circle.

Angle $DCE = 47°$ and angle $CEA = 85°$.

Find the values of $w$, $x$ and $y$.




$w$ = .....

$x$ = .....

$y$ = .....

[3]

*[June/2018/P23/Q20]*

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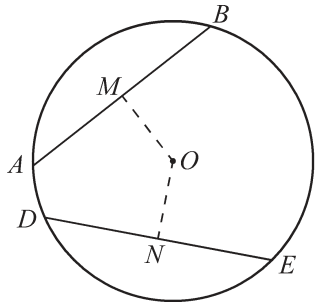
**3.** The diagram shows a circle, centre $O$.

$AB$ and $DE$ are chords of the circle.
$M$ is the mid-point of $AB$ and $N$ is the mid-point of $DE$.
$AB = DE = 9$ cm and $OM = 5$ cm.

Find $ON$.




$ON$ = ..... cm [1]

*[Nov/2018/P22/Q2]*

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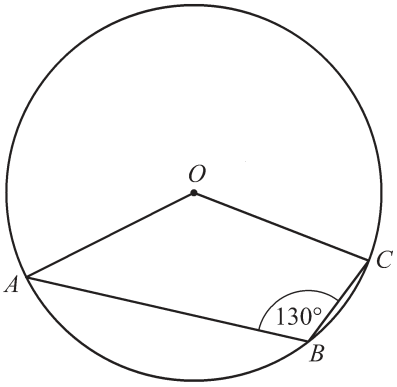
**4.** $A$, $B$ and $C$ are points on the circle, centre $O$.

Find the obtuse angle $AOC$.




Angle $AOC$ = ..... [2]

*[June/2019/P21/Q8]*

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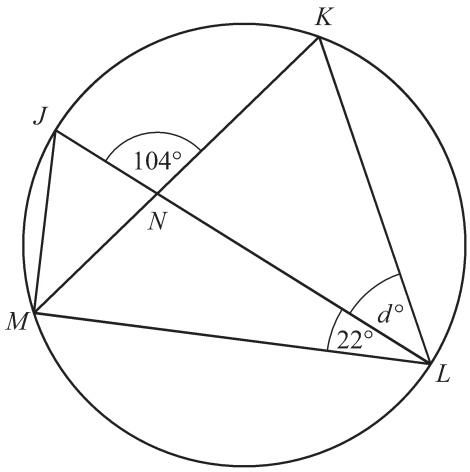
**5.** $J$, $K$, $L$ and $M$ are points on the circumference of a circle with diameter $JL$.

$JL$ and $KM$ intersect at $N$.
Angle $JNK = 104°$ and angle $MLJ = 22°$.

Work out the value of $d$.




$d$ = ..... [4]

*[June/2019/P23/Q19]*

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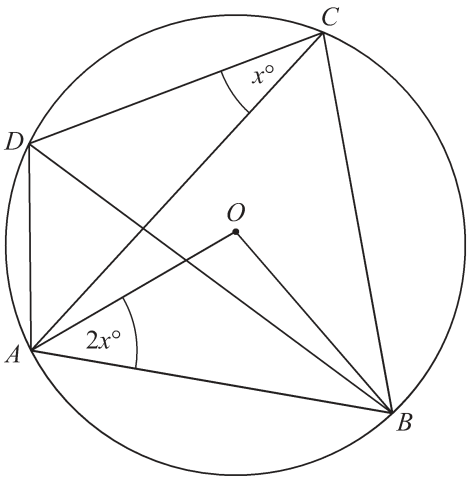
**6.** In the diagram, $A$, $B$, $C$ and $D$ lie on the circumference of a circle, centre $O$.

Angle $ACD = x°$ and angle $OAB = 2x°$.

Find an expression, in terms of $x$, in its simplest form for




(a) angle $AOB$,

Angle $AOB$ = ..... [1]

(b) angle $ACB$,

Angle $ACB$ = ..... [1]

(c) angle $DAB$.

Angle $DAB$ = ..... [2]

*[Nov/2019/P22/Q19]*

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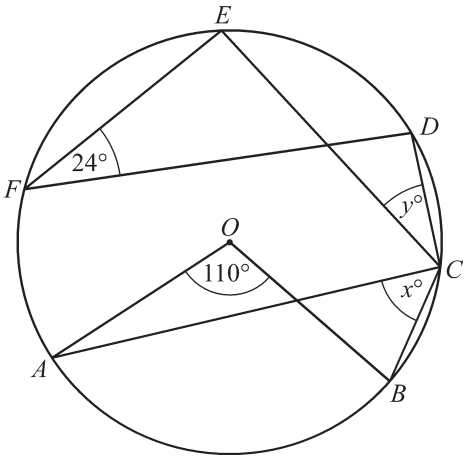
**7.** Points $A$, $B$, $C$, $D$, $E$ and $F$ lie on the circle, centre $O$.

Find the value of $x$ and the value of $y$. [2]



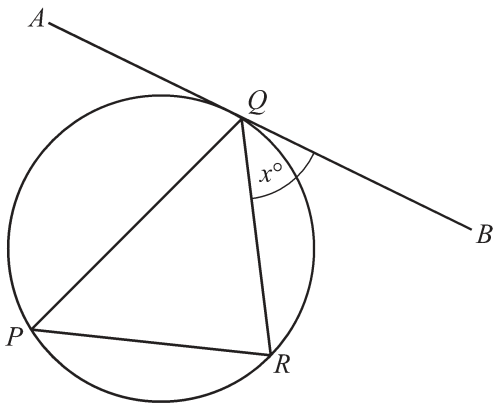

$x$ = ....., $y$ = .....

*[June/2020/P21/Q10]*

**8.** *P*, *R* and *Q* are points on the circle.

*AB* is a tangent to the circle at *Q*.
*QR* bisects angle *PQB*.
Angle *BQR* = $x°$ and $x < 60$.

Use this information to show that triangle *PQR* is an isosceles triangle.
Give a geometrical reason for each step of your work.




[3]

*[June/2020/P21/Q15]*

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**9.** *A*, *B*, *C* and *D* lie on the circle, centre *O*.

*TA* is a tangent to the circle at *A*.
Angle *ABC* = 131° and angle *ADB* = 20°.

Find

(a) angle *ADC*,

Angle *ADC* = ..... [1]

(b) angle *AOC*,

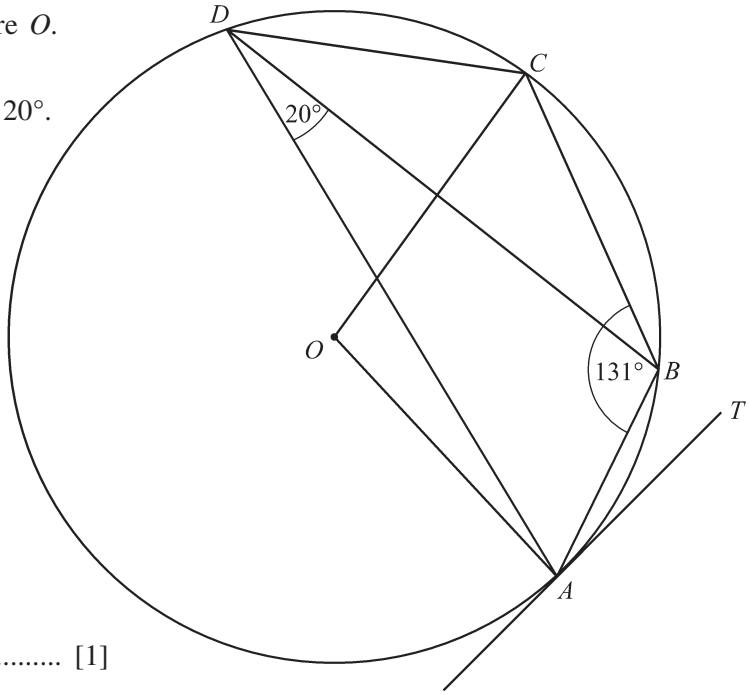
Angle *AOC* = ..... [1]

(c) angle *BAT*,

Angle *BAT* = ..... [1]

(d) angle *OAB*.

Angle *OAB* = ..... [1]




*[June/2020/P22/Q20]*

# SOLUTIONS

## Topic 27 - Circle Theorems

**1.** $AM = \frac{12}{2} = 6$ cm

In $\Delta OAM$, using Pythagoras Theorem,

$OA = \sqrt{OM^2 + AM^2}$

$\Rightarrow \quad OA = \sqrt{4^2 + 6^2}$

$\Rightarrow \quad OA = \sqrt{16+36} = \sqrt{52}$

$\therefore$ Radius of the circle $= \sqrt{52}$ cm.

**2.** $ABCE$ is a cyclic quadrilateral,

$\therefore \quad w° + 85° = 180°$ (opp. $\angle$s of a cyclic quad. are supplementary)

$\Rightarrow \quad w° = 180° - 85°$

$= 95°$

$ABCD$ is a cyclic quadrilateral,

$\therefore \quad x° + w° = 180°$

$\Rightarrow \quad x° + 95° = 180°$

$\Rightarrow \quad x° = 180° - 95°$

$= 85°$

$x° + y° + 47° = 180°$ ($\angle$ sum of a triangle is 180°)

$\Rightarrow \quad 85° + y° + 47° = 180°$

$\Rightarrow \quad y° = 180° - 132°$

$= 48°$

> Note:
> $x°$ can also be found as below.
> $A\hat{D}C = A\hat{E}C$ (angles in the same segment)
> $\Rightarrow \quad x° = 85°$

**3.** $ON = 5$ cm (equal chords are equidistant from centre).

**4.** Reflex $A\hat{O}C = 2(130°) = 260°$

($\angle$ at centre is $2\times\angle$ at circumference)

$\therefore$ Obtuse $A\hat{O}C = 360° - 260°$

$= 100°$

**5.** In $\Delta JML$,

$J\hat{M}L = 90°$ (right angle in semicircle)

$M\hat{J}L + 90° + 22° = 180°$ (angle sum of a $\Delta$ is 180°)

$\Rightarrow \quad M\hat{J}L = 180° - 90° - 22°$

$\Rightarrow \quad M\hat{J}L = 68°$

$M\hat{K}L = M\hat{J}L$ (angles in the same segment)

$\Rightarrow \quad M\hat{K}L = 68°$

Now, in $\Delta KLN$,

$d° + 68° = 104°$ (ext. angle of $\Delta$ = sum of opp. interior angles)

$\Rightarrow \quad d° = 104° - 68°$

$= 36°$

**6.** **(a)** $\Delta OAB$ is isosceles with $OA = OB$ (radii of circle),

$O\hat{B}A = 2x°$ (base angles of isosceles $\Delta$)

$\therefore \quad A\hat{O}B + 2x° + 2x° = 180°$

$\Rightarrow \quad A\hat{O}B = 180° - 4x°$

**(b)** $A\hat{C}B = \frac{1}{2}(A\hat{O}B)$ ($\angle$ at centre is $2\times\angle$ at circumference)

$\Rightarrow \quad A\hat{C}B = \frac{1}{2}(180° - 4x°)$

$= 90° - 2x°$

**(c)** $D\hat{C}B = D\hat{C}A + A\hat{C}B$

$= x° + 90° - 2x° = 90° - x°$

$ABCD$ is a cyclic quadrilateral

$\Rightarrow \quad D\hat{A}B + D\hat{C}B = 180°$ (opp. $\angle$s of a cyclic quad. add up to 180°)

$\Rightarrow \quad D\hat{A}B + 90° - x° = 180°$

$\Rightarrow \quad D\hat{A}B = 180° - 90° + x°$

$= 90° + x°$

***Alternative Solution***

$D\hat{B}A = D\hat{C}A = x°$ (angles in the same segment)

$A\hat{D}B = A\hat{C}B$ (angles in the same segment)

$= 90° - 2x°$

$\therefore \quad D\hat{A}B + A\hat{D}B + D\hat{B}A = 180°$ ($\angle$ sum of a $\Delta$)

$\Rightarrow \quad D\hat{A}B + 90° - 2x° + x° = 180°$

$\Rightarrow \quad D\hat{A}B + 90° - x° = 180°$

$\Rightarrow \quad D\hat{A}B = 180° - 90° + x°$

$= 90° + x°$

**7.** $x° = \frac{110°}{2}$ ($\angle$ at centre is $2\times\angle$ at circumference)
$= 55°$

$y° = 24°$ (angles in the same segment)

**8.** Given that, $QR$ bisects angle $PQB$,

$\Rightarrow \quad P\hat{Q}R = B\hat{Q}R = x°$

Also, $Q\hat{P}R = x°$ (alternate segment theorem)

$\therefore$ $\Delta PQR$ has two equal angles.

Given that, $x° < 60°$, therefore the triangle is not an equilateral triangle.

Thus, $\Delta PQR$ is an isosceles triangle.

**9.** **(a)** $ABCD$ is a cyclic quadrilateral

Opposite angles of a cyclic quadrilateral are supplementary

$\therefore \quad A\hat{D}C + 131° = 180°$

$A\hat{D}C = 180° - 131°$

$= 49°.$

**(b)** $A\hat{O}C = 2(A\hat{D}C)$ ($\angle$ at centre is $2\times\angle$ at circumference)

$\Rightarrow \quad A\hat{O}C = 2(49°)$

$= 98°.$

**(c)** $B\hat{A}T = 20°$ (alternate segment theorem)

**(d)** $O\hat{A}T = 90°$ (radius $\perp$ tangent)

$\therefore \quad O\hat{A}B = 90° - B\hat{A}T$

$= 90° - 20°$

$= 70°$

**10.** Opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \quad 2x° + x° + 60° = 180°$

$\Rightarrow \quad 3x° = 120° \quad \Rightarrow \quad x° = 40°$

Also, $y° + 4x° - 87° = 180°$

$\Rightarrow \quad y° + (4)(40°) - 87° = 180°$

$\Rightarrow \quad y° + 160° - 87° = 180°$

$\Rightarrow \quad y° + 73° = 180° \quad \Rightarrow \quad y° = 180° - 73° = 107°$

**11.** $O\hat{A}D = O\hat{C}D = 90°$ (radius $\perp$ tangent)

$\therefore \quad A\hat{O}C = 180° - 44°$

$= 136°$

$x° = \frac{1}{2}(A\hat{O}C)$ ($\angle$ at centre is $2\times\angle$ at circumference)

$\Rightarrow \quad x° = \frac{1}{2}(136°) = 68°$

**12.** **(a)** $P\hat{Q}T = 50°$ (alternate segment theorem)

$\Delta PQT$ is isosceles with $PT = PQ$

$\Rightarrow \quad P\hat{T}Q = P\hat{Q}T = 50°$ (base $\angle$ of isosceles $\Delta$)

$\therefore \quad T\hat{P}Q = 180° - 50° - 50°$

$= 180° - 100°$

$= 80°$

**(b)** $w = 68°$

Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle

$3x° + 2x° = 180°$

$5x° = 180°$

$x° = \frac{180°}{5} = 36°$

Opposite angles of a cyclic quadrilateral are supplementary

**13.** **(a)** $x = 55°$ because, alternate segment theorem.

**(b)** Tangents drawn from an external point to a circle are equal in lengths.

Therefore, $SV = SR$.

Thus $\Delta SVR$ is isosceles.

**14.** $x° = 38°$ (alternate segment theorem)

$A\hat{C}B = x°$ (alternate angles)

$A\hat{B}C + 60° = 180°$ (opp. $\angle$s of a cyclic quadrilateral add to 180°)

$\Rightarrow \quad A\hat{B}C = 180° - 60° = 120°$

In $\Delta ABC$,

$y° + x° + A\hat{B}C = 180°$ (angle sum of a $\Delta$)

$\Rightarrow \quad y° + 38° + 120° = 180°$

$\Rightarrow \quad y° = 180° - 38° - 120°$

$= 22°$

**15.** Diameter of circle = $DE$

Angle subtended by diameter at any point on the circumference is 90°.

In $\Delta DEF$, $D\hat{F}E = 180° - 82° - 8° = 90°$.

Thus $DE$ is the diameter.