REDSPOT

IGCSE

(Syllabus 0580)

MATHEMATICS

Paper 4 (Extended) - All Variants

(Topical)

period	**2018 to June-2024**
contents	**June & November, Paper 4 (P41, P42 & P43) Worked Solutions**
form	**Topic By Topic**
compiled for	**IGCSE**

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TOPIC 10

Solutions of Equations

1. (a) $s = ut + \frac{1}{2}at^2$

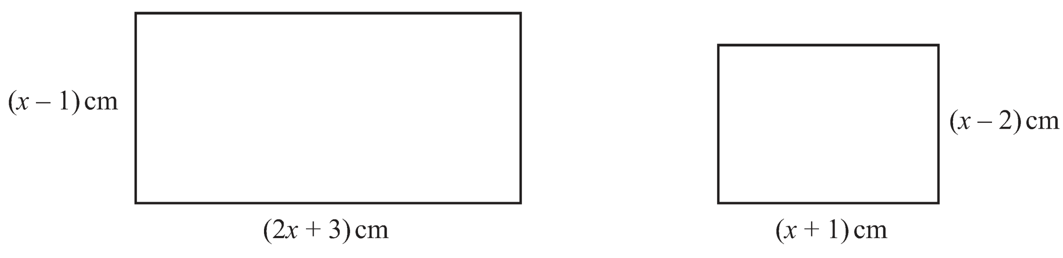
(i) Find s when $t = 26.5$, $u = 104.3$ and $a = -2.2$.
Give your answer in standard form, correct to 4 significant figures.

s = ... [4]

(ii) Rearrange the formula to write a in terms of u, t and s.

a = ... [3]

(b)



The difference between the areas of the two rectangles is 62 cm^2.

(i) Show that $x^2 + 2x - 63 = 0$.

[3]

(ii) Factorise $x^2 + 2x - 63$.

.... [2]

(iii) Solve the equation $x^2 + 2x - 63 = 0$ to find the difference between the perimeters of the two rectangles.

.... cm [2]

[June/2019/P41/Q7]

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2. Solve the equation. $3(x-4) + \dfrac{x+2}{5} = 6$

x = ... [4]

[June/2019/P42/Q6(c)]

3. (a) Oranges cost 21 cents each.

Alex buys x oranges and Bobbie buys $(x + 2)$ oranges.

The total cost of these oranges is \$4.20 .

Find the value of x.

x = ... [3]

(b) The cost of one ruler is r cents.

The cost of one protractor is p cents.

The total cost of 5 rulers and 1 protractor is 245 cents.

The total cost of 2 rulers and 3 protractors is 215 cents.

Write down two equations in terms of r and p and solve these equations to find the cost of one protractor.

.... cents [5]

(c) Carol walks 12 km at x km/h and then a further 6 km at $(x – 1)$ km/h.

The total time taken is 5 hours.

(i) Write an equation, in terms of x, and show that it simplifies to $5x^2 - 23x + 12 = 0$.

[3]

(ii) Factorise $5x^2 - 23x + 12$.

.... [2]

(iii) Solve the equation $5x^2 - 23x + 12 = 0$.

x = ... or x = ... [1]

(iv) Write down Carol's walking speed during the final 6 km.

.... km/h [1]

[Nov/2019/P41/Q7]

---

4. Make p the subject of

(i) $5p + 7 = m$,

p = ... [2]

(ii) $y^2 - 2p^2 = h$.

p = ... [3]

[Nov/2019/P42/Q8(a)]

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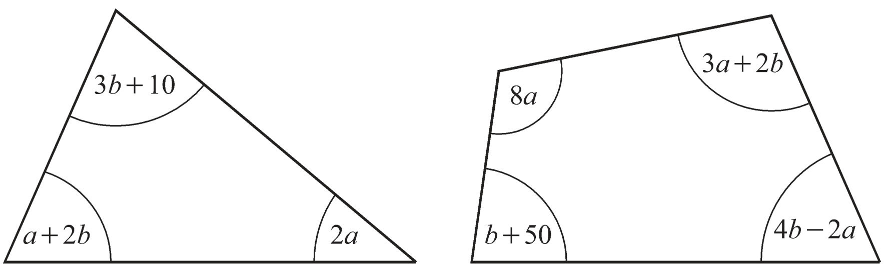
5. (a) The diagram shows a triangle and a quadrilateral.
All angles are in degrees.



(i) For the triangle, show that $3a+5b=170$.

[1]

(ii) For the quadrilateral, show that $9a+7b=310$.

[1]

(iii) Solve these simultaneous equations.
Show all your working.

a = ...

b = ...
[3]

(iv) Find the size of the smallest angle in the triangle.

.... [1]

(b) Solve the equation $6x-3=-12$.

x = ... [2]

(c) Rearrange $2(4x-y)=5x-3$ to make y the subject.

y = ... [3]

[Nov/2019/P43/Q2(a,b,c)]

6. Solve $\dfrac{1}{x}-\dfrac{2}{x+1}=3$

Show all your working and give your answers correct to 2 decimal places.

x = ... or x = ... [7]

[Nov/2019/P43/Q10]

---

7. (a) $s=ut+\frac{1}{2}at^2$

Find the value of s when u = 5.2, t = 7 and a = 1.6.

s = ... [2]

(b) Solve

(i) $\dfrac{15}{x}=-3$

x = ... [1]

(ii) $4(5-3x)=23$

x = ... [3]

[June/2020/P41/Q3(a,c)]

SOLUTIONS

Topic 10 - Solutions of Equations

1. **(a)** **(i)** $s = (104.3)(26.5) + \frac{1}{2}(-2.2)(26.5)^2$

$= 2763.95 - 772.475$

$= 1991.475 \approx 1.991 \times 10^3$

(ii) $s = ut + \frac{1}{2}at^2$

$\Rightarrow \frac{1}{2}at^2 = s - ut$

$\Rightarrow at^2 = 2s - 2ut \quad \Rightarrow \quad a = \frac{2s - 2ut}{t^2}$

(b) **(i)** Area of big rectangle $= (2x+3)(x-1)$

$= 2x^2 - 2x + 3x - 3$

$= 2x^2 + x - 3$

Area of small rectangle $= (x+1)(x-2)$

$= x^2 - 2x + x - 2$

$= x^2 - x - 2$

Given that, difference in the areas of the two rectangles = 62 cm^2

$\Rightarrow (2x^2 + x - 3) - (x^2 - x - 2) = 62$

$\Rightarrow 2x^2 + x - 3 - x^2 + x + 2 - 62 = 0$

$\Rightarrow x^2 + 2x - 63 = 0$

(ii) $x^2 + 2x - 63$

$= x^2 + 9x - 7x - 63$

$= x(x+9) - 7(x+9)$

$= (x-7)(x+9)$

(iii) $x^2 + 2x - 63 = 0$

$\Rightarrow (x-7)(x+9) = 0$

$\Rightarrow x = 7$ or $x = -9$ (rejected due to -ve sign)

Using $x = 7$, we have,

Perimeter of larger rectangle

$= 2(2x + 3 + x - 1)$

$= 2\left(2(7) + 3 + 7 - 1\right) = 2(23) = 46$ cm

Perimeter of smaller rectangle

$= 2(x + 1 + x - 2)$

$= 2(7 + 1 + 7 - 2) = 2(13) = 26$ cm

$\therefore$ Difference between perimeters

$= 46 - 26 = 20$ cm

2. $3(x-4) + \frac{x+2}{5} = 6$

$\Rightarrow 3x - 12 + \frac{x+2}{5} = 6$

$\Rightarrow \frac{5(3x) - 5(12) + x + 2}{5} = 6$

$\Rightarrow 15x - 60 + x + 2 = 30$

$\Rightarrow 16x - 88 = 0$

$\Rightarrow 16x = 88 \quad \Rightarrow \quad x = \frac{88}{16} = 5.5$

3. **(a)** $21(x) + 21(x+2) = 420$

$21x + 21x + 42 = 420$

$42x = 378$

$x = 9$

(b) $5r + p = 245$ .... (1)

$2r + 3p = 215$ .... (2)

Solving the equations simultaneously,

eq. (1)×2: $\quad 10r + 2p = 490$

eq. (2)×5: $\quad \underline{-10r \; - 15p = -1075}$

$-13p = -585$

$\Rightarrow \quad p = \frac{-585}{-13} = 45$

$\therefore$ Cost of one protractor = 45 cents

(c) **(i)** Time taken for 1st part of journey $= \frac{12}{x}$ h

Time taken for 2nd part $= \frac{6}{x-1}$ h

Given that, $\frac{12}{x} + \frac{6}{x-1} = 5$

$\Rightarrow \frac{12(x-1) + 6x}{x(x-1)} = 5$

$\Rightarrow 12x - 12 + 6x = 5x(x-1)$

$\Rightarrow 18x - 12 = 5x^2 - 5x$

$\Rightarrow 5x^2 - 23x + 12 = 0$

(ii) $5x^2 - 23x + 12$

$= 5x^2 - 20x - 3x + 12$

$= 5x(x-4) - 3(x-4)$

$= (x-4)(5x-3)$

(iii) $5x^2 - 23x + 12 = 0$

$\Rightarrow \ (x-4)(5x-3) = 0$

$\Rightarrow \ x - 4 = 0 \quad \text{or} \quad 5x - 3 = 0$

$\Rightarrow \ x = 4 \quad \text{or} \quad x = \frac{3}{5}$

(iv) Carol's walking speed $= x - 1$

$= 4 - 1 = 3$ km/h

4. (a) (i) $5p + 7 = m$

$5p = m - 7 \quad \Rightarrow \quad p = \frac{m-7}{5}$

(ii) $y^2 - 2p^2 = h$

$\Rightarrow \ y^2 - h = 2p^2$

$\Rightarrow \ p^2 = \frac{y^2 - h}{2} \quad \Rightarrow \quad p = \pm\sqrt{\frac{y^2 - h}{2}}$

5. (a) (i) $(a + 2b) + (2a) + (3b + 10) = 180°$

$3a + 5b + 10 = 180$

$3a + 5b = 170$

(ii) $(b + 50) + (4b - 2a) + (3a + 2b) + (8a)$

$= 360°$

$\Rightarrow \ 9a + 7b + 50 = 360$

$\Rightarrow \ 9a + 7b = 310$

(iii) $3a + 5b = 170$ .... (1)

$9a + 7b = 310$ .... (2)

eq. (1)×3: $\quad 9a + 15b = 510$

eq. (2): $\quad \underline{9a + 7b = 310}$ (−)

$8b = 200 \ \Rightarrow \ b = 25$

Substitute $b = 25$ into eq. (1),

$3a + 5(25) = 170$

$3a + 125 = 170$

$3a = 45 \quad \Rightarrow \quad a = 15$

(iv) Smallest angle in the triangle $= 2a$

$= 2(15) = 30°$

(b) $6x - 3 = -12$

$\Rightarrow \ 6x = -12 + 3$

$\Rightarrow \ 6x = -9 \quad \Rightarrow \quad x = -\frac{9}{6} = -1.5$

(c) $2(4x - y) = 5x - 3$

$\Rightarrow \ 8x - 2y = 5x - 3$

$\Rightarrow \ 8x - 5x + 3 = 2y$

$\Rightarrow \ 2y = 3x + 3 \quad \Rightarrow \quad y = \frac{3x+3}{2}$

6. $\frac{1}{x} - \frac{2}{x+1} = 3$

$\Rightarrow \ \frac{x + 1 - 2x}{x(x+1)} = 3$

$\Rightarrow \ \frac{1 - x}{x^2 + x} = 3$

$\Rightarrow \ 1 - x = 3(x^2 + x)$

$\Rightarrow \ 1 - x = 3x^2 + 3x$

$\Rightarrow \ 3x^2 + 4x - 1 = 0$

Using quadratic formula,

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(3)}$

$= \frac{-4 \pm \sqrt{28}}{6}$

$\Rightarrow \ x = \frac{-4 + \sqrt{28}}{6} \quad \text{or} \quad x = \frac{-4 - \sqrt{28}}{6}$

$= 0.215 \qquad\qquad = -1.549$

$\therefore \ x = 0.22 \quad \text{or} \quad x = -1.55$

7. (a) $s = ut + \frac{1}{2}at^2$

$\Rightarrow \ s = (5.2)(7) + \frac{1}{2}(1.6)(7)^2$

$= 75.6$

(b) (i) $\frac{15}{x} = -3$

$\Rightarrow \ 15 = -3x$

$\Rightarrow \ x = \frac{15}{-3} = -5$

(ii) $4(5 - 3x) = 23$

$20 - 12x = 23$

$12x = 20 - 23$

$x = -\frac{3}{12} = -\frac{1}{4}$

8. (a) (i) $x^2 + 8x - 9$

$= x^2 + 8x + (4)^2 - (4)^2 - 9$

$= (x + 4)^2 - 16 - 9$

$= (x + 4)^2 - 25$

(ii) $x^2 + 8x - 9 = 0$

$\Rightarrow \ (x + 4)^2 - 25 = 0$

$\Rightarrow \ (x + 4)^2 = 25$

$\Rightarrow \ x + 4 = \pm 5$

$\Rightarrow \ x + 4 = 5 \quad \text{or} \quad x + 4 = -5$

$\therefore \ x = 1 \quad \text{or} \quad x = -9$

(b) Given solution is, $x=\dfrac{-7\pm\sqrt{61}}{2}$

Comparing it with, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$-b=-7 \quad\Rightarrow\quad b=7$

Also, $b^2-4ac=61$

$\Rightarrow\quad (7)^2-4(1)(c)=61$

$\Rightarrow\quad 49-4c=61$

$\Rightarrow\quad 4c=-12 \quad\Rightarrow\quad c=-3$

$\therefore\quad b=7,\quad c=-3$

9. (a) $\dfrac{2x+5}{3-x}=\dfrac{14}{15}$

$15(2x+5)=14(3-x)$

$30x+75=42-14x$

$44x=-33 \quad\Rightarrow\quad x=-\dfrac{33}{44} \quad\Rightarrow\quad x=-\dfrac{3}{4}$

(b) $y=4-x$ ....(1) $\quad x^2+2y^2=67$........(2)

Substitute eq. (1) into eq. (2)

$\Rightarrow\quad x^2+2(4-x)^2=67$

$\Rightarrow\quad x^2+2(16-8x+x^2)=67$

$\Rightarrow\quad x^2+32-16x+2x^2-67=0$

$\Rightarrow\quad 3x^2-16x-35=0$

$\Rightarrow\quad 3x^2-21x+5x-35=0$

$\Rightarrow\quad 3x(x-7)+5(x-7)=0$

$\Rightarrow\quad (x-7)(3x+5)=0$

$\Rightarrow\quad x=7 \quad$ or $\quad x=-\dfrac{5}{3}$

Subst. $x=7$ into (1), $\quad y=4-7=-3$

Subst. $x=-\dfrac{5}{3}$ into (1),

$\Rightarrow\quad y=4-\left(-\dfrac{5}{3}\right) \quad\Rightarrow\quad y=4+\dfrac{5}{3}=\dfrac{17}{3}$

$\therefore\quad x=7,\quad y=-3,$ and $\quad x=-\dfrac{5}{3},\quad y=\dfrac{17}{3}$

10. (a) $FE=2x-(x+3)=x-3$ cm

Total area of two rectangles $=342$

$\Rightarrow\quad (4x-5)(x+3)+(x+1)(x-3)=342$

$\Rightarrow\quad 4x^2+7x-15+x^2-2x-3=342$

$\Rightarrow\quad 5x^2+5x-18=342$

$\Rightarrow\quad 5x^2+5x-360=0 \quad$ (divide by 5)

$\Rightarrow\quad x^2+x-72=0$

(b) $x^2+x-72=0$

$\Rightarrow\quad x^2+9x-8x-72=0$

$\Rightarrow\quad x(x+9)-8(x+9)=0$

$\Rightarrow\quad (x+9)(x-8)=0$

$\therefore\quad x=-9$ or $x=8.$

(c) Note that, $CD+EF=2x$

$AF+ED=4x-5$

Perimeter $=2x+(4x-5)+2x+(4x-5)$

$=12x-10$

Using $x=8$ from part (b),

Perimeter $=12(8)-10$

$=96-10=86$ cm.

(d) In $\Delta BCD,\quad \tan D\hat{B}C=\dfrac{CD}{BC}$

$\Rightarrow\quad \tan D\hat{B}C=\dfrac{x+3}{4x-5}$

$\Rightarrow\quad \tan D\hat{B}C=\dfrac{8+3}{4(8)-5}$

$\Rightarrow\quad \tan D\hat{B}C=\dfrac{11}{27} \quad\Rightarrow\quad D\hat{B}C\approx 22.2°$

11. $\dfrac{2}{x}=\dfrac{6}{2-x}$

$\Rightarrow\quad 2(2-x)=6x$

$\Rightarrow\quad 4-2x=6x \quad\Rightarrow\quad 8x=4 \quad\Rightarrow\quad x=\dfrac{1}{2}$

12. Line : $\quad y=3x+2$ .... (1)

Curve : $\quad y=2x^2+7x-11$ .... (2)

Substitute line into curve,

$3x+2=2x^2+7x-11$

$\Rightarrow\quad 2x^2+4x-13=0$

Using quadratic formula,

$$x=\frac{-4\pm\sqrt{(4)^2-4(2)(-13)}}{2(2)}$$

$$=\frac{-4\pm\sqrt{120}}{4}$$

$\Rightarrow\quad x=\dfrac{-4+\sqrt{120}}{4} \quad$ or $\quad x=\dfrac{-4-\sqrt{120}}{4}$

$\Rightarrow\quad x=1.74 \quad$ or $\quad x=-3.74$

Substitute $x=1.74$ into eq.(1),

$y=3(1.74)+2=7.22$

Substitute $x=-3.74$ into eq.(1),

$y=3(-3.74)+2=-9.22$

$\therefore\quad$ Coordinates of points of intersection are,

$(1.74,\ 7.22)$ and $(-3.74,\ -9.22)$

TOPIC 24

Mensuration

1. (a)

The diagram shows a hemispherical bowl of radius 5.6 cm and a cylindrical tin of height 10 cm.

(i) Show that the volume of the bowl is 368 cm^3, correct to the nearest cm^3.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[2]

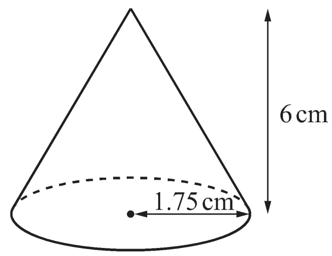
(ii) The tin is completely full of soup.

When all the soup is poured into the empty bowl, 80% of the volume of the bowl is filled.

Calculate the radius of the tin.

.... cm [4]

(b) The diagram shows a cone with radius 1.75 cm and height 6 cm.

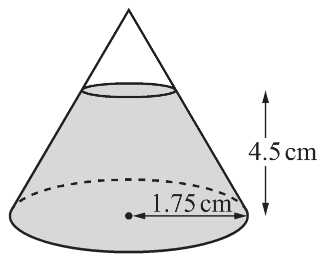


(i) Calculate the total surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

.... cm^2 [5]

(ii) The cone contains salt to a depth of 4.5 cm.

The top layer of the salt forms a circle that is parallel to the base of the cone.



(a) Show that the volume of the salt inside the cone is $18.9\ cm^3$, correct to 1 decimal place.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

[4]

(b) The salt is removed from the cone at a constant rate of 200 mm^3 per second.

Calculate the time taken for the cone to be completely emptied.
Give your answer in seconds, correct to the nearest second.

.... s [3]

[June/2019/P43/Q4]

2. (a) (i) Calculate the **external curved** surface area of a cylinder with radius 8 m and height 19 m.

.... m^2 [2]

(ii) This surface is painted at a cost of \$0.85 per square metre.

Calculate the cost of painting this surface.

\$ .... [2]

(b) A solid metal sphere with radius 6 cm is melted down and all of the metal is used to make a solid cone with radius 8 cm and height h cm.

(i) Show that $h = 13.5$.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

[2]

(ii) Calculate the slant height of the cone.

.... cm [2]

(iii) Calculate the curved surface area of the cone.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

.... cm^2 [1]

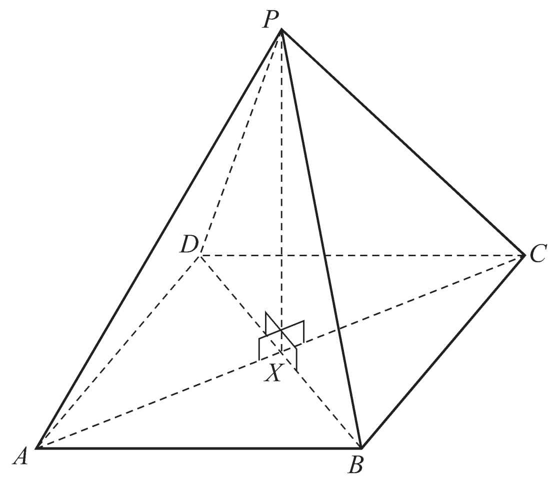
(c) Two cones are mathematically similar.
The total surface area of the smaller cone is 80 cm^2.
The total surface area of the larger cone is 180 cm^2.
The volume of the smaller cone is 168 cm^3.

Calculate the volume of the larger cone.

.... cm^3 [3]

(d) The diagram shows a pyramid with a square base $ABCD$.
$DB = 8$ cm.
P is vertically above the centre, X, of the base and $PX = 5$ cm

Calculate the angle between PB and the base $ABCD$..

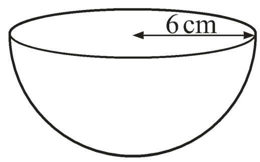


.... [3]

[Nov/2019/P41/Q4]

3. The diagram shows a hemisphere with radius 6 cm.
Calculate the volume. Give the units of your answer.

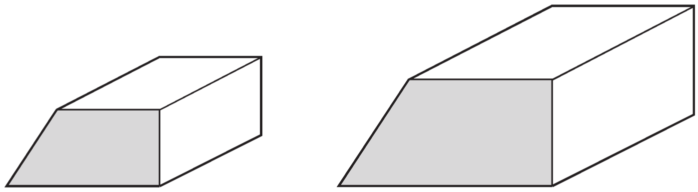
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]



.... [3]

[Nov/2019/P42/Q4(a)]

4.



The diagram shows two mathematically similar solid metal prisms.
The volume of the smaller prism is 648 cm^3 and the volume of the larger prism is 2187 cm^3.
The area of the cross-section of the smaller prism is 36 cm^2.

(i) Calculate the area of the cross-section of the larger prism.

.... cm^2 [3]

(ii) The larger prism is melted down into a sphere.

Calculate the radius of the sphere.

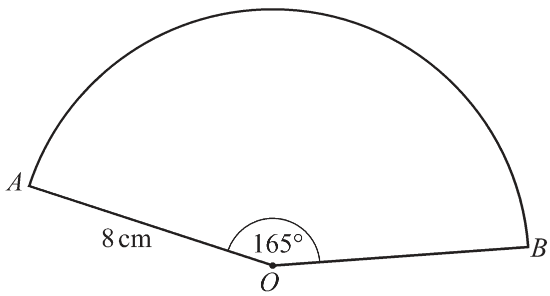
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

.... cm [3]

[Nov/2019/P43/Q6(b)]

5. The diagram shows a sector of a circle with centre O, radius 8 cm and sector angle 165°.

(a) Calculate the total perimeter of the sector.



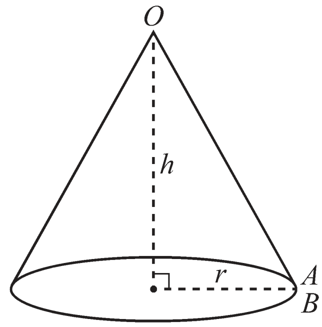
.... cm [3]

(b) The surface area of a sphere is the same as the area of the sector.
Calculate the radius of the sphere.
[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

.... cm [4]

(c) A cone is made from the sector by joining OA to OB.

(i) Calculate the radius, r, of the cone.



r = ... cm [2]

(ii) Calculate the volume of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

.... cm^3 [4]

[June/2020/P41/Q9]

6. (a) A cylinder with radius 6 cm and height h cm has the same volume as a sphere with radius 4.5 cm.

Find the value of h.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

h = ... [3]

(b) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres, each of radius r cm.

Find the value of r.

r = ... [3]

(c) A solid cylinder has radius x cm and height $\frac{7x}{2}$ cm.

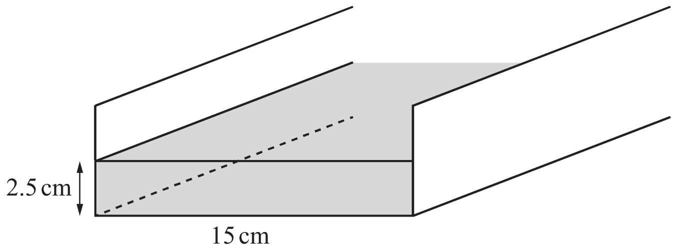
The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder. Find an expression for R in terms of x.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

R = ... [3]

[June/2020/P42/Q8(b,c,d)]

7.



Water flows at a speed of 20 cm/s along a rectangular channel into a lake.
The width of the channel is 15 cm.
The depth of the water is 2.5 cm.
Calculate the amount of water that flows from the channel into the lake in 1 hour.
Give your answer in litres.

.... litres [4]

[June/2020/P43/Q6(b)]

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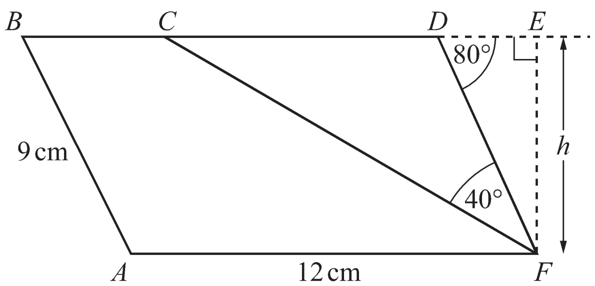
8. (a)



ABDF is a parallelogram and *BCDE* is a straight line.
$AF = 12$ cm, $AB = 9$ cm, angle $CFD = 40°$ and angle $FDE = 80°$.

(i) Calculate the height, h, of the parallelogram.

h = ... cm [2]

SOLUTIONS

Topic 24 - Mensuration

1. (a) (i) Volume of bowl $= \frac{1}{2}\left(\frac{4}{3}\pi(5.6)^3\right)$

$= 367.809 \approx 368\text{ cm}^3$

(ii) Volume of tin = 80% of volume of bowl

$\Rightarrow \quad \pi r^2(10) = \frac{80}{100}\times 368$

$\Rightarrow \quad 10\pi r^2 = 294.4$

$\Rightarrow \quad r^2 = \frac{294.4}{10\pi}$

$\Rightarrow \quad r^2 = 9.371 \quad \Rightarrow \quad r = 3.06$ cm.

(b) (i) Using Pythagoras theorem, slant height of the cone is, $l = \sqrt{1.75^2 + 6^2}$

$= \sqrt{39.0625} = 6.25$ cm

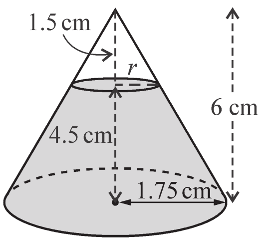
Total surface area of the cone

= area of base + curved surface area

$= \pi(1.75)^2 + \pi(1.75)(6.25)$

$= 9.621 + 34.36 = 43.981 \approx 44.0\text{ cm}^2$

(ii) (a)



Let radius of top smaller cone be r cm

Height of smaller cone $= 6 - 4.5$

$= 1.5$ cm

Using rule of similar triangles,

$\frac{r}{1.75} = \frac{1.5}{6}$

$\Rightarrow \quad r = \frac{1.5}{6}\times 1.75 = 0.4375$ cm

Volume of salt = vol. of larger cone − vol. of smaller cone

$= \frac{1}{3}\pi(1.75)^2(6) - \frac{1}{3}\pi(0.4375)^2(1.5)$

$= 19.242 - 0.3007 \approx 18.9\text{ cm}^3$ (to 1 dp)

(b) Volume of sand in $\text{mm}^3 = 18.9\times 10^3$

$= 18900\text{ mm}^3$

Rate of removing sand $= 200\text{ mm}^3/\text{s}$

$\therefore$ Time taken to empty the cone

$= \frac{18900}{200} = 94.5\text{ s} \approx 95$ s.

2. (a) (i) Curved surface area $= 2\pi rh$

$= 2\pi(8)(19) \approx 955\text{ m}^2$

(ii) Cost of painting $= \$0.85\times 955$

$= \$811.75$

(b) (i) Volume of sphere melted = volume of cone

$\Rightarrow \quad \frac{4}{3}\pi(6)^3 = \frac{1}{3}\pi(8)^2 h$

$\Rightarrow \quad 288\pi = \frac{64}{3}\pi h$

$\Rightarrow \quad h = 288\pi\times\frac{3}{64\pi} = 13.5$ cm.

(ii) Let slant height be l.

Using Pythagoras theorem,

$l = \sqrt{8^2 + 13.5^2}$

$= \sqrt{246.25} \approx 15.7$ cm

(iii) Curved surface area

$= \pi(8)(15.7)$

$= 394.58 \approx 395\text{ cm}^2$

(c) $\frac{\text{Area}_{\text{small}}}{\text{Area}_{\text{large}}} = \left(\frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}}\right)^2$

$\Rightarrow \quad \frac{80}{180} = \left(\frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}}\right)^2$

$\Rightarrow \quad \frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}} = \sqrt{\frac{80}{180}}$

$\Rightarrow \quad \frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}} = \frac{2}{3}$

Now, $\frac{\text{Volume}_{\text{small}}}{\text{Volume}_{\text{large}}} = \left(\frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}}\right)^3$

$\Rightarrow \quad \frac{168}{\text{Volume}_{\text{large}}} = \left(\frac{2}{3}\right)^3$

$\Rightarrow \quad \frac{168}{\text{Volume}_{\text{large}}} = \frac{8}{27}$

$\Rightarrow \quad 168\times 27 = 8(\text{Volume}_{\text{large}})$

$\Rightarrow \quad \text{Volume}_{\text{large}} = \frac{168\times 27}{8} = 567\text{ cm}^3$

(d) In ΔPBX, the angle between PB and the base $ABCD$ is $P\hat{B}X$

$$XB = \frac{1}{2}DB = \frac{1}{2}(8) = 4 \text{ cm}$$

Now, $\tan P\hat{B}X = \dfrac{PX}{XB}$

$$\Rightarrow \tan P\hat{B}X = \frac{5}{4} \Rightarrow P\hat{B}X = 51.3°$$

3. $$\text{Volume} = \frac{1}{2}\left(\frac{4}{3}\pi(6)^3\right) = 452.39 \approx 452 \text{ cm}^3$$

4. **(i)** $$\frac{\text{Volume}_{\text{large}}}{\text{Volume}_{\text{small}}} = \left(\frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}}\right)^3$$

$$\Rightarrow \frac{2187}{648} = \left(\frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}}\right)^3$$

$$\Rightarrow \frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}} = \sqrt[3]{\frac{2187}{648}}$$

$$\Rightarrow \frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}} = \frac{3}{2}$$

Now, $\dfrac{\text{Area}_{\text{large}}}{\text{Area}_{\text{small}}} = \left(\dfrac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}}\right)^2$

$$\Rightarrow \frac{\text{Area}_{\text{large}}}{36} = \left(\frac{3}{2}\right)^2$$

$$\Rightarrow \text{Area}_{\text{large}} = \frac{9}{4}\times 36 = 81 \text{ cm}^2$$

(ii) Volume of sphere = volume of large prism

$$\frac{4}{3}\pi r^3 = 2187$$

$$r^3 = 2187\times\frac{3}{4\pi}$$

$$r^3 = 522.108$$

$$r \approx 8.05 \text{ cm}$$

5. **(a)** Perimeter of sector = Arc length $+ OA + OB$

$$= \frac{165°}{360°}\times 2\pi(8) + 8 + 8$$

$$= 23.04 + 16$$

$$= 39.04 \approx 39.0 \text{ cm}$$

(b) Surface area of sphere = area of the sector

$$\Rightarrow 4\pi r^2 = \frac{165°}{360°}\times(\pi)(8)^2$$

$$\Rightarrow r^2 = \frac{165°}{360°}\times(\pi)(8)^2\times\frac{1}{4\pi}$$

$$\Rightarrow r^2 = 7.333 \Rightarrow r = 2.71 \text{ cm.}$$

(c) **(i)** Circumference of base of cone = arc length of sector

$$\Rightarrow 2\pi r = \frac{165°}{360°}\times 2(\pi)(8)$$

$$\Rightarrow r = \frac{165°}{360°}\times(8) \Rightarrow r = 3.67$$

$\therefore$ Radius of the cone = 3.67 cm

(ii) By pythagoras theorem, height of cone is, $h = \sqrt{(8)^2 - (3.67)^2}$

$$= \sqrt{50.5311} = 7.11 \text{ cm}$$

Volume of cone $= \dfrac{1}{3}\pi(3.67)^2(7.11)$

$$= 100.28 \approx 100 \text{ cm}^3$$

6. **(a)** Volume of cylinder = volume of sphere

$$\Rightarrow \pi(6)^2 h = \frac{4}{3}(\pi)(4.5)^3$$

$$\Rightarrow 36\pi h = 121.5\pi$$

$$\Rightarrow h = \frac{121.5\pi}{36\pi} = 3.375 \text{ cm}$$

(b) Volume of 40 spheres = Volume of cube

$$\Rightarrow 40\left(\frac{4}{3}\pi r^3\right) = 20^3$$

$$\Rightarrow \frac{160}{3}\pi r^3 = 8000$$

$$\Rightarrow r^3 = 8000\times\frac{3}{160\pi}$$

$$\Rightarrow r^3 = 47.746 \Rightarrow r = 3.63 \text{ cm}$$

(c) Surface area of sphere = total surface area of cylinder

$$\Rightarrow 4\pi R^2 = 2\pi x^2 + 2\pi(x)\left(\frac{7x}{2}\right)$$

$$\Rightarrow 4\pi R^2 = 2\pi x^2 + 7\pi x^2$$

$$\Rightarrow 4\pi R^2 = 9\pi x^2$$

$$\Rightarrow R^2 = \frac{9\pi x^2}{4\pi} \Rightarrow R = \sqrt{\frac{9x^2}{4}} = \frac{3x}{2} \text{ cm.}$$

7. Speed of water flow = 20 cm/s

Area of cross section $= 15\times 2.5 = 37.5 \text{ cm}^2$

Volume of water flowing into the lake in 1 second $= 37.5\times 20 = 750 \text{ cm}^3$

$\therefore$ Amount of water that flows in 1 hour

$$= 750\times 60\times 60$$

$$= 2700\,000 \text{ cm}^3$$

$$= \frac{2700\,000}{1000} = 2700 \text{ litres}$$

8. (a) (i) In ΔDEF, $FD = 9$ cm

$\sin 80° = \dfrac{h}{9} \Rightarrow h = 9\sin 80° = 8.86$ cm

(ii) $D\hat{C}F + 40° = 80°$ (ext. $\angle$ of Δ = sum of opp. interior angles)

$\Rightarrow D\hat{C}F = 80° - 40° = 40°$

As, $C\hat{F}D = D\hat{C}F = 40°$ (two equal $\angle$s)

$\therefore$ ΔCDF is an isosceles triangle

(iii) $CD = DF = 9$ cm (ΔCDF is isosceles)

$\Rightarrow BC = 12 - 9 = 3$ cm

Area of trapezium $ABCF$

$= \dfrac{1}{2}(12+3)(8.86) = 66.45 \text{ cm}^2$

(b) $A\hat{D}C = 90°$ (right angle in semicircle)

$A\hat{C}D = 21°$ (angles in the same segment)

Now, in ΔACD, $\cos 21° = \dfrac{12}{AC}$

$\Rightarrow AC = \dfrac{12}{\cos 21°} = 12.85$ cm

Radius of circle, $r = \dfrac{12.85}{2} = 6.425$ cm

$\therefore$ Area of circle $= \pi(6.425)^2$

$= 129.687 \approx 130 \text{ cm}^2$

(c) Perimeter of square = perimeter of sector

$\Rightarrow 4 \times 8 = 9.5 + 9.5 + \text{arc length of sector}$

$\Rightarrow 32 = 19 + \dfrac{x°}{360°} \times 2\pi(9.5)$

$\Rightarrow 32 = 19 + \dfrac{19\pi x°}{360°}$

$\Rightarrow 13 = \dfrac{19\pi x°}{360°}$

$\Rightarrow x° = 13 \times \dfrac{360°}{19\pi} \Rightarrow x° = 78.4°$

9. (a) Volume of cuboid $= 8 \times 5 \times 11$

$= 440 \text{ cm}^3$

(b) We can decide this by finding the length of the diagonal AG.

Using Pythagoras theorem on ΔABC,

$AC^2 = 8^2 + 5^2 = 89$ cm

Again by Pythagoras theorem on ΔAGC,

$AG = \sqrt{AC^2 + 11^2}$

$= \sqrt{89 + 11^2} = \sqrt{210} = 14.5$ cm

$\therefore$ Yes, pencil fits completely inside the cuboid.

(c) (i) In ΔABC, $\tan C\hat{A}B = \dfrac{5}{8}$

$\Rightarrow C\hat{A}B = \tan^{-1}\left(\dfrac{5}{8}\right) = 32.0°$

(ii) From (b), $AC = \sqrt{89} = 9.434$ cm

In ΔAGC, $\tan G\hat{A}C = \dfrac{11}{AC}$

$\Rightarrow G\hat{A}C = \tan^{-1}\left(\dfrac{11}{9.434}\right) = 49.4°$

10. (a) Total surface area of cone = total surface area of hemisphere

$\Rightarrow \pi(2.4)^2 + \pi(2.4)(6.3) = \pi R^2 + \dfrac{1}{2}(4\pi R^2)$

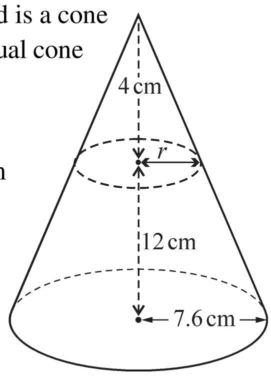
$\Rightarrow 5.76\pi + 15.12\pi = 3\pi R^2$

$\Rightarrow 20.88\pi = 3\pi R^2$

$\Rightarrow R^2 = \dfrac{20.88\pi}{3\pi}$

$\Rightarrow R^2 = 6.96 \Rightarrow R = 2.64$ cm

(b) The top section removed is a cone that is similar to the actual cone



$\dfrac{r}{7.6} = \dfrac{4}{16}$

$\Rightarrow r = \dfrac{4}{16} \times 7.6 = 1.9$ cm

$\therefore$ Radius of top section cone = 1.9 cm

Vol. of remaining solid

= Vol. of actual cone − Vol. of top section cone

$= \dfrac{1}{3}\pi(7.6)^2(16) - \dfrac{1}{3}(\pi)(1.9)^2(4)$

$= 967.78 - 15.12 = 952.66 \approx 953 \text{ cm}^3$

11. (a) In ΔABC, by Pythagoras theorem,

$BC = \sqrt{20^2 - 13^2} = \sqrt{231} \approx 15.2$ cm

Total surface area

$= 2\left(\dfrac{1}{2}(13)(15.2)\right) + (20 \times 24) + (24 \times 15.2) + (13 \times 24)$

$= 197.6 + 480 + 364.8 + 312$

$= 1354.4 \text{ cm}^2 \approx 1350 \text{ cm}^2$

(b) Volume = area of triangle × prism length

$= \dfrac{1}{2}(13)(15.2) \times 24$

$= 2371.2 \approx 2370 \text{ cm}^3$