To provide an idea about what this book contains, only few pages taken randomly from the book are shown here.

GCE 'O' Level Additional Mathematics (Topical)

CONTENTS

Topic 1

Set Language and Notation

1 *(J06/P2/Q6)*

Questions are not shown in Preview

Question 1

Thinking Process

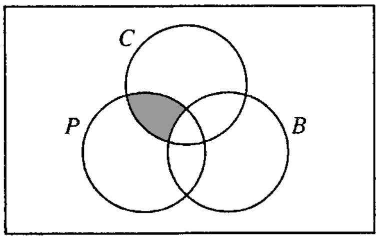
(a) (i) Shade the region where the circle representing Physics and Chemistry overlap but out of the circle representing Biology.

(ii) Shade the entire region that is out of the circle representing Physics but in the circle representing Chemistry and the circle representing Biology.

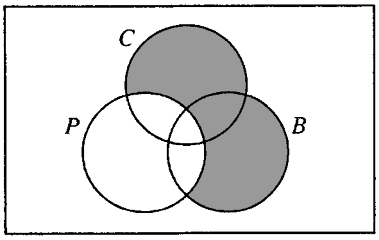
(b) Draw the 2 cases of subsets and disjoint sets.

Solution

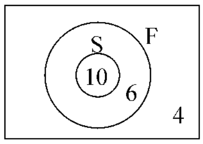
(a) (i) ℰ



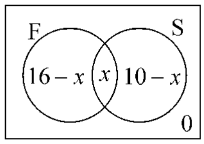
(ii) ℰ



(b) ε = 20



ε = 20



$$16 - x + x + 10 - x = 20$$
$$26 - x = 20$$
$$x = 6$$

(i) Maximum value of $n(F \cap S) = 10$

Minimum value of $n(F \cap S) = 6$

(ii) Maximum value of $n(F \cup S) = 20$

Minimum value of $n(F \cup S) = 16$

2 *(D06/P1/Q1)*

Questions are not shown in Preview

Question 2

Thinking Process

(i) The symbol for 'not an element' is $\notin$.

(ii) The elements that are not in set *B* is denoted by *B*'.

(iii) *C* and *D* are disjoint sets.

Solution *with* TEACHER's COMMENTs

(i) $x \notin A$.

(ii) $n(B') = 16$.

(iii) $C \cap D = \phi$.

$C \cap D = \phi$ but $n(C \cap D) = 0$. Do not write $C \cap D = 0$.

3 *(J05/P2/Q8)*

Questions are not shown in Preview

Question 3

Thinking Process

(i) Since $n(C \cap D) = k$,

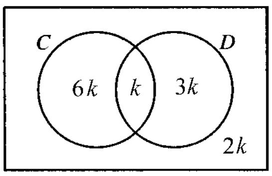
$n(C) = 7 \times n(C \cap D) \Rightarrow n(C \cap D') = 7k - k = 6k$.

Similarly, $n(D \cap C') = 4k - k = 3k$.

(ii) Given that $n(C' \cap D') = 165\ 000$, find $n(\varepsilon)$.

Solution

(i) $\mathscr{E} = 12k$



$n(C \cap D') = 7k - k$
$= 6k$

$n(D \cap C') = 4k - k$
$= 3k$

(ii) Given $n(C' \cap D') = 165\ 000$
$\Rightarrow n(\varepsilon) = 6 \times 165\ 000$
$= 990\ 000$

4 *(D05/P1/Q2)*

Questions are not shown in Preview

Question 4

Thinking Process

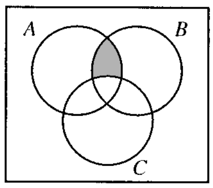
(a) (i) The whole set of *B* which is out of *A*.
(ii) The whole set of *B* with the region out of *A*.

(b) (i) The intersection of *A* and *B* but out of *C*.
(ii) The whole of *B* and *C* but out of *A*.

Solution

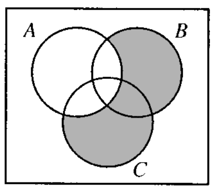
(a) (i) $A' \cap B$.
(ii) $B \cup A'$.

(b) (i) $\mathscr{E}$



$A \cap B \cap C'$

(ii) $\mathscr{E}$



$A' \cap (B \cup C)$

Topic 16

Differentiation and Applications

1 *(J06/P1/Q1)*

Questions are not shown in Preview

Question 1

Thinking Process

Differentiate the equation to find $\frac{dy}{dx}$ and substitute $x = 2$ into $\frac{dy}{dx}$ to find gradient.

Solution

$$y = (x-1)(2x-3)^8$$

$$\frac{dy}{dx} = (x-1)\cdot 8(2x-3)^7\cdot(2) + (2x-3)^8\cdot(1)$$
$$= (2x-3)^7\cdot[16(x-1)+(2x-3)]$$
$$= (2x-3)^7\cdot(16x-16+2x-3)$$
$$= (2x-3)^7\cdot(18x-19)$$

At $x = 2$, gradient of curve
$$= [2(2)-3]^7\cdot[18(2)-19]$$
$$= 36-19$$
$$= 17$$

2 *(J06/P2/Q1)*

Questions are not shown in Preview

Question 2

Thinking Process

Find $\frac{dy}{dx}$.

Apply related rate of change: $\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$.

Solution

$$y = (3x-1)\cdot\ln x$$

$$\frac{dy}{dx} = (3x-1)\cdot\left(\frac{1}{x}\right) + (\ln x)\cdot(3)$$
$$= \frac{3x-1}{x} + 3\ln x$$

At $x = 1$, $\frac{dy}{dx} = \frac{3-1}{1} + 3\ln 1$
$$= 2$$

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$$
$$= 2\times 3$$
$$= 6$$

$\therefore$ The rate of increase of y when $x = 1$ is 6 units / sec.

3 *(J06/P2/Q4 a)*

Question 3

Thinking Process

(a) $\frac{d}{dx}e^{f(x)} = f'(x)\cdot e^{f(x)}$.

Solution

(a) $\frac{d}{dx}e^{\tan x} = \sec^2 x\cdot e^{\tan x}$.

4 *(J06/P2/Q9)*

Questions are not shown in Preview

Question 4

Thinking Process

(i) Let 120 = total surface area of the cuboid and make h the subject of the formula.

(ii) Find the volume of the cuboid and substitute the expression for h into the formula for V.

(iii) Equate $\frac{4x}{3}$ to the expression of h found in (i). Find $\frac{dV}{dx}$ and substitute the value of x in and show that $\frac{dV}{dx} = 0$.

Solution

(i) $2(2x)(x) + 2xh + 2(2x)(h) = 120$

$\Rightarrow 2x^2 + xh + 2hx = 60$

$\Rightarrow 2x^2 + 3hx = 60$

$\Rightarrow 3hx = 60 - 2x^2$

$\Rightarrow h = \frac{60-2x^2}{3x}$

(ii) $V = (2x)(x)(h)$

$= 2x^2\left(\frac{60-2x^2}{3x}\right)$

$= 2x^2\left(\frac{20}{x}-\frac{2x}{3}\right)$

$= 40x - \frac{4x^3}{3}$ (shown)

(iii) $h = \frac{4x}{3}$

$\Rightarrow \frac{4x}{3} = \frac{60-2x^2}{3x}$

$\Rightarrow 12x^2 = 180 - 6x^2$

$\Rightarrow 18x^2 = 180$

$x^2 = 10$

$V = 40x - \frac{4x^3}{3}$

$\frac{dv}{dx} = 40 - 4x^2$

At $x^2 = 10$, $\frac{dv}{dx} = 40 - 4(10)$

$= 0$ (shown)

$\therefore V$ has a stationary value when $h = \frac{4x}{3}$.

5 *(D06/P1/Q3)*

Questions are not shown in Preview

Question 5

Thinking Process

(i) Find $\frac{dy}{dx}$. Substitute $x = 2$ into $\frac{dy}{dx}$.

(ii) Find δy given that $\delta y \approx \frac{dy}{dx} \cdot \delta x$.

Solution

(i) $y = \frac{8}{(3x-4)^2}$

$y = 8(3x-4)^{-2}$

$\frac{dy}{dx} = -16(3x-4)^{-3} \cdot (3)$

$= \frac{-48}{(3x-4)^3}$

At $x = 2$, $\frac{dy}{dx} = \frac{-48}{(6-4)^3}$

$= \frac{-48}{8}$

$= -6$

$\therefore$ Gradient of curve at $x = 2$ is -6.

(ii) $\delta y \approx \frac{dy}{dx} \cdot \delta x$

$\approx -6p$

6 *(D06/P1/Q9)*

Questions are not shown in Preview

Question 6

Thinking Process

(i) Apply quotient rule to find $\frac{dy}{dx}$. Show $\frac{dy}{dx} \neq 0$.

(ii) Find co-ord. of P and Q.

Solution

(i) $y = \frac{2x-4}{x+3}$

$\frac{dy}{dx} = \frac{(x+3)(2)-(2x-4)}{(x+3)^2}$

$= \frac{2x+6-2x+4}{(x+3)^2}$

$= \frac{10}{(x+3)^2}$

Since $(x+3)^2 \geq 0$, $\Rightarrow \frac{10}{(x+3)^2} > 0$, the curve has no turning points.

(ii) At $y = 0$, $\frac{2x-4}{x+3} = 0$

$\Rightarrow 2x - 4 = 0$

$\Rightarrow x = 2$

P is (2, 0)

At $P(2, 0)$, $\frac{dy}{dx} = \frac{10}{(2+3)^2}$

$= \frac{10}{25}$

$= \frac{2}{5}$

Gradient of tangent $= \frac{2}{5}$

$y - 0 = \frac{2}{5}(x-2)$

$y = \frac{2}{5}x - \frac{4}{5}$ (equation of tangent)

At $x = 0$, $y = -\frac{4}{5}$

$\therefore Q\left(0, -\frac{4}{5}\right)$

$\therefore$ Area of $\Delta POQ = \frac{1}{2} \times (2) \times \left(\frac{4}{5}\right)$

$= \frac{4}{5}$ sq. units

November 2009

PAPER 2

✐ **means " before that, do this ! "**

Answer **all** *the questions.*

1 *Topic: 2*

Questions are not shown in Preview

Question 1

Thinking Process

(i) To find range ✐ find f(x) for $x > 0$.

(ii) To find f^{-1} ✐ let $y = f(x)$ and express x in terms of y.

(iii) ✐ Domain of $f^{-1}(x)$ = Range of f(x).

Solution

(i) Range of f: $f(x) > \frac{1}{e}$ **Ans.**

(ii) $f(x) = e^{x-1}$

Let $f(x) = y$

$$\Rightarrow \quad y = e^{x-1}$$
$$\ln y = \ln e^{x-1}$$
$$\ln y = x - 1$$
$$x = 1 + \ln|y| \qquad \because x = f^{-1}(y)$$
$$\therefore \quad f^{-1}(y) = 1 + \ln|y|$$

or $f^{-1}(x) = 1 + \ln|x|$ **Ans.**

(iii) Domain of $f^{-1}(x)$ = range of $f(x)$

$\therefore$ domain of f^{-1}: $x > \frac{1}{e}$ **Ans.**

2 *Topic: 12*

Questions are not shown in Preview

Question 2

Thinking Process

(i) Expand using binomial theorem for up to four terms.

(ii) Expand $\left(2-\frac{x}{2}\right)^6$ up to the term x^3. Multiply $(1+x)^2$ and $\left(2-\frac{x}{2}\right)^6$ up to term x^3.

Solution

(i) $\left(2-\frac{x}{2}\right)^6$

$$= 2^6 + {}^6C_1(2)^5\left(-\frac{x}{2}\right)^1 + {}^6C_2(2)^4\left(-\frac{x}{2}\right)^2 + {}^6C_3(2)^3\left(-\frac{x}{2}\right)^3 + \ldots$$

$$= 64 + 6(32)\left(-\frac{x}{2}\right) + 15(16)\left(\frac{x^2}{4}\right) + 20(8)\left(-\frac{x^3}{8}\right) + ..$$

$$= 64 - 96x + 60x^2 - 20x^3 + \ldots \quad \textbf{Ans.}$$

(ii) $(1+x)^2\left(2-\frac{x}{2}\right)^6$

$$= (1+2x+x^2)(64 - 96x + 60x^2 - 20x^3 + \ldots.)$$
$$= \ldots\ldots.. - 20x^3 + 120x^3 - 96x^3 + \ \ldots\ldots..$$
$$= \ldots\ldots..\ 4x^3 + \ \ldots\ldots..$$

$\therefore$ coefficient of $x^3 = 4$ **Ans.**

3 *Topic: 8*

Questions are not shown in Preview

Question 3

Thinking Process

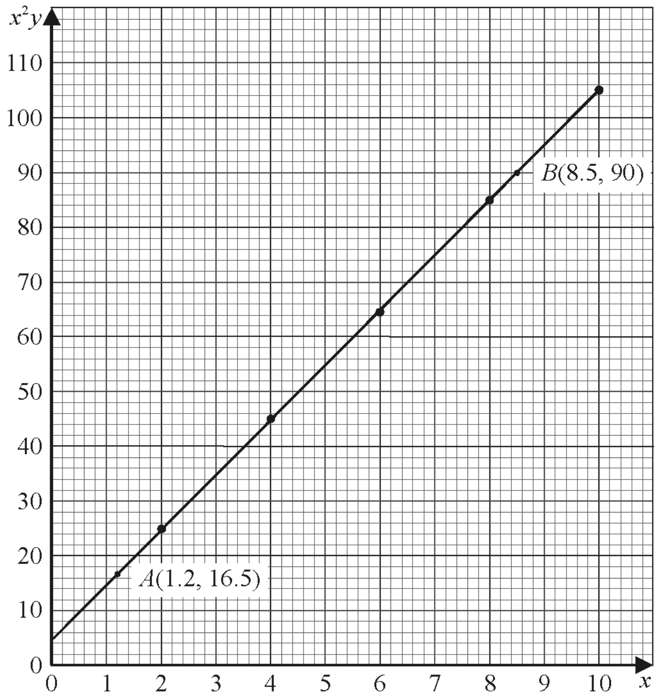
(i) Form a new table for x against x^2y and draw the graph.

(ii) To find a and b ✐ rewrite the equation in linear form: $Y = mX + C$.

Solution

(i)

x	2	4	6	8	10
y	6.24	2.82	1.79	1.33	1.05
x^2y	24.96	45.12	64.44	85.12	105



(ii) $y = \dfrac{a}{x^2} + \dfrac{b}{x}$

$\Rightarrow \quad x^2y = a + bx$

$\Rightarrow \quad x^2y = b(x) + a$ ....(1)

from graph, using two points, $A(1.2, 16.5)$ and

$B(8.5, 90)$, gradient $= \dfrac{90 - 16.5}{8.5 - 1.2} = \dfrac{73.5}{7.3} = 10.1$

from equation (1), gradient $= b$

$\therefore \quad b = 10.1$ **Ans.**

from equation (1), y-intercept $= a$

from graph, y-intercept $= 5$

$\therefore \quad a = 5$ **Ans.**

4 *Topic: 16*

Question 4

Thinking Process

To find stationary points ✐ find $\dfrac{dy}{dx}$ and equate it to 0 to solve for stationary points.

To find the nature ✐ evaluate $\dfrac{d^2y}{dx^2}$.

If $\dfrac{d^2y}{dx^2} < 0$ at the given value of x, y is maximum.

If $\dfrac{d^2y}{dx^2} > 0$, y is minimum.

Solution

$y = x^3 + 3x^2 - 45x + 60$

$\dfrac{dy}{dx} = 3x^2 + 6x - 45$

For stationary points, $\dfrac{dy}{dx} = 0$

$\Rightarrow \quad 3x^2 + 6x - 45 = 0$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$\therefore \quad x = -5, \quad \text{or} \quad x = 3$

When $x = -5$, $\quad y = (-5)^3 + 3(-5)^2 - 45(-5) + 60$

$= -125 + 75 + 225 + 60 = 235$

When $x = 3$, $\quad y = (3)^3 + 3(3)^2 - 45(3) + 60$

$= 27 + 27 - 135 + 60 = -21$

$\therefore$ coordinates of the stationary points are $(-5,\ 235)$ and $(3,\ -21)$ **Ans.**

Differentiating $\dfrac{dy}{dx}$ w.r.t. x.

$\dfrac{d^2y}{dx^2} = 6x + 6$

When $x = -5$, $\quad \dfrac{d^2y}{dx^2} = 6(-5) + 6 = -24 < 0$

$\therefore \quad (-5,\ 235)$ is a maximum point on the curve. **Ans.**

When $x = 3$, $\quad \dfrac{d^2y}{dx^2} = 6(3) + 6 = 24 > 0$

$\therefore \quad (3,\ -21)$ is a minimum point on the curve. **Ans.**